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ter verkrijging van de graad van doctor aan de Katholieke Universiteit Brabant, op gezag van de rector magnificus, prof. dr. L.F.W. de Klerk, in het openbaar te verdedigen ten overstaan van een door het college van dekanen aangewezen commissie in de aula van de Universiteit op maandag 16 december 1996 om 16.15 uur door

Christiaan Lambert Jan Peter van Raalte

geboren op 22 november 1967 te Utrecht

PROMOTOR: Prof. dr. P.H.M. Ruys

COPROMOTOR: Dr. R.P. Gilles

Ter nagedachtenis van Wim de Laat

LIEDJE

Lieg alsjeblieft niet tegen me
niet over iets groots niet over iets
anders. Liever hoor ik het
vernietigendste dan dat je liegt
want dat is nog vernietigender.

Lieg niet over liefde,
iets dat je voelt of iets dat je
zou willen voelen. Liever word ik
bedroefd dan dat je liegt
want dat is nog bedroevender.

Lieg niet tegen me over gevaar
want ik voel toch je angst
en wat ik gewaar word is waar
of ik ken je niet en dat
is nog gevaarlijker.

Lieg niet tegen me over ziekte
liever kijk ik die diepte in
dan dat ik mij verlies in één
van jouw lieve verzinsels
want daarmee verlies ik me dieper.

Lieg niet tegen me over sterven
want zo lang we er nog zijn
vind ik dat toegangsloze
niet mededelen wat je denkt
erger en zo veel doder.

Judith Herzberg

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Chris van Raalte
October 1996

Contents

Chapter 1

Introduction

The organization of markets is an important field of inquiry in modern economics. Who organizes markets, and how, is the question this monograph attempts to shed some light on. In persuance thereof, we develop and analyze models of, first, formation and selection of local, costly competitive markets (Chapters 2, 3 and 4), and second, models of bilateral matching with intermediation (Chapter 5).

We illustrate the idea of market formation with a metaphorical example of an agricultural economy of farmers who grow crops. The farmers may remain autarkic, that is, consume their own crop. They prefer, however, variety in consumption, and different farmers grow different kinds of crops, so there is an incentive to meet other farmers and trade crops. Finding trading partners is a costly and difficult task for the farmers. They will probably have to visit several other farmers before finding trading partners. And, even then, an agreement may not be reached. So coordination is needed to make trade viable. Farmers are willing to pay up to a certain amount, depending on the gains-from-trade offered by them, for the establishment and running of coordinating institutions.

Certain farmers recognize the need for coordination, and the willingness to pay for it by the other farmers. They respond by setting up (central) 'market places', at which they enable the other farmers to trade crops. Providing a market place is costly for these 'middlemen' in two respects. First, they incur a *set-up cost*; a proportion of their crops has to be invested; Markets take time to organize. Another, physical interpretation is that a farmer builds a market place on his land. Secondly, farmers incur an *opportunity cost*; middlemen's crops cannot be traded. Again, this could be caused by time constraints. This means that the intermediating farmers stand entirely outside the market, so in fact cease being farmers. To trade off set-up and opportunity costs, middlemen obtain fees in the form of parts of crops of the farmers trading through them. As long as fees are not too large, and there are not too many middlemen, the economy is in a state in which all farmers are better off than in an autarky.

We could imagine a collection of market places evolving as follows. The economy starts in a state of autarky, every farmer consuming his own crop. After some time, some farmers decide to set up a market place as a middleman. Other farmers are attracted to these market places. Since the middlemen form a small group in this early stage, their payoffs are relatively large. Other farmers observe this and enter the market as middleman also. This process of *market formation* goes on until the position of middleman becomes less attractive because of the large number of middlemen already active. Then, farmers will have a stronger tendency to become traders, and a process of *market selection* starts, in which traders select among the most favorable market places. The selection criterion for a trader is the terms-of-trade of a market, that is, the utility of his competitive equilibrium bundle resulting from the net endowments of all traders present at that market, compared to the utility of his gross endowment. The formation of markets by middlemen and selection of markets by traders is analyzed by means of the *market formation game*[1]. A static analysis of the game is performed in Chapters 2 and 3. A dynamic model which formalizes the idea of evolving markets described above is given in Chapter 4. It should be noted, that the process of market formation and selection described in this paragraph is presented as a sequential process, whereas actually the two processes largely happen simultaneously.

The second part (Chapter 5) of the monograph is concerned with *bilateral matching* through matchmakers. In the model of Chapter 5, there are two types of consumers. Every agent seeks a match with an agent from the other type. A successful match gives both agents a surplus. Examples are real estate markets, where buyers and sellers of houses are matched, and marriage markets, where men and women are matched. Typical for the markets in these examples is the presence of matchmakers; in real estate markets,

[1] A game-theoretic approach of economic problems has become increasingly popular. See, e.g. van Damme (1995)).

buyers and sellers are matched by brokers.

Matchmakers exist because it is costly, difficult, or even impossible for market sides to establish matches on their own account. A reason may be a high degree of heterogeniety in tastes among agents; for each agent, only a small number of other agents yields a potential successful match. The marriage market is an illustration: the search for the 'ideal' partner on one's own account poses difficulties because of informational problems. Match makers take advantage of the need for intermediation by charging a commission fee to both market sides. In the bilateral matching model there is discrimination between market sides in fees, and a fee is charged only if a successful match is found. In general, the number of consumers of the one type is different from the number of consumers of the other type. Therefore, matchmakers have an incentive to attract consumers on the short side of the market by 'subsidizing' them; That is, only matched consumers are charged a fee, so that the short side determines a matchmaker's payoff.

The market formation and bilateral matching model are both models of costly intermediation by middlemen, who do not interfere with the activities of market participants. In the market formation model, middlemen activate competitive markets, making trade possible for traders. In the bilateral matching model, middlemen match buyers and sellers. The models differ in the following aspects. First, the type of market they describe. The market formation model describes Walrasian markets, on which traders are matched through an anonymous market price. In the bilateral matching model, consumers are matched non-anonymously, and no uniform market price is generated. Secondly, the role of middlemen. In the market formation model, the coalition of middlemen is determined endogenously by the market formation game, whereas in the bilateral matching model, it is given exogenously. On could say there is a 'market' for middlemen in the first model, since the middleman's job is accessible to all, unlike in the second model.

The remainder of this introductory chapter is organized as follows. In section 1, a survey of models on commodity exchange is given. Section 2 considers the public aspects of market organization. Section 3 provides a survey of evolutionary models which is expanded in Chapter 4. Finally, Section 4 gives an overview of this monograph.

1.1 Organization of commodity exchange

The mainstream models of exchange economies consider *barter* economies, and *market* economies. In barter models, starting with Edgeworth (1881), consumers trade directly with each other. Edgeworth introduced the recontracting principle, by which consumers will retrade commodities until no improvement is possible. Recontracting yields the Core, containing all allocations robust against recontracting. In market models, initiated by Walras (1874), consumers do not trade directly, but through an anonymous competitive market. Based on market prices, consumers reveal their supply and demand. The market is in equilibrium when aggregate supply equals aggregate demand. Debreu and Scarf (1963), and Hildenbrand (1974) show that if the number of consumers tends to infinity, the Core of a barter economy shrinks to the set of competitive equilibria in a market economy.

A major drawback of the Walrasian general equilibrium model is that there is no explicit institution that determines market prices. In the absence of some form of organization, demand and supply do not interact to yield equilibrium, an idea captured in the tri-polar model of Ruys (1974). The problem of absent price setting institutions is sometimes solved by adding the Walrasian auctioneer to the economy, as for example in Arrow and Debreu (1954), and Debreu (1959). The auctioneer is assumed to be a price setting agent who takes the quantities the other agents in the economy want to buy and sell as given and sets his prices for 'the market' in order to maximize his payoffs from these trades. The other agents act as price takers. An equilibrium in the economy is a Nash equilibrium of the game with a price-setting auctioneer and price-taking consumers.

The auctioneer is an implicit agent; he is not modelled explicitly. The market formation models in the first part of this monograph incorporate explicit 'auctioneers'. Namely, *middlemen* are introduced, who set up costly, local competitive markets for *traders*. Middlemen are Arrow/Debreu-type consumers, that give up trading their own endowments. They then share similarities with the Walrasian auctioneer, in that they are not part of the economy, just as the auctioneer in the Arrow/Debreu model. So, in that sense middlemen may be considered as explicit auctioneers.

The market formation model is based on the *Separation Principle*, which isolates *individual* from *institutional* characteristics. The individual sphere is formed by consumers with their endowments and utility functions, whereas the institutional sphere is given by a collection of local, costly competitive markets organized by middlemen. A market formation game connects the individual and the institutional sphere by assigning consumers to markets and roles. *A priori*, consumers have no institutional characteristics. A different approach is the *Interdependency Principle*, according to which agents have *a priori* institutional characteristics. This principle is applied to models of *relationally structured economies*, in which there exist communication links between agents[2]. A characterization of agents in these type of economies is given by Gilles and Ruys (1990). Gilles, Haller and Ruys (1994) examine their Core properties (see also Gilles, Haller and Ruys (1997)). Characteristic for these models is that communication links are non-hierarchical, that is, no agent has a dominance relation over another. Van den Brink (1994) studies hierarchical relations between economic agents.

On the level of the individual, the market formation model follows the *atomistic approach* due to Hildenbrand (1974): Every single consumer has zero measure and is therefore 'powerless'. Zero fractions of middlemen are powerless in the sense that they are not able to activate markets, and zero fractions of traders are not able to trade. Aumann (1964) proposes the *mechanistic approach* instead, under which individual consumers have arbitrary small, but positive, measure. A 'large' economy lies at the basis of the market formation model, and therefore *coalitions* of positive measure should be considered. The observation that coalitions are relevant in a large economy is followed by Gilles (1990), who takes *primitive coalitions* as actors of the economy. Primitive coalitions have the ability to improve upon proposed commodity allocations. In an example (Gilles (1990, Example 7.4.1, page 248)), primitive coalitions necessarily contain groups of 'managers' in order for consumers to trade. In the market formation model, such a primitive coalition is analogous to a coalition of traders and middlemen. Middlemen are needed to activate a market and facilitate trade for traders. In Chapters 2 and 3, the behavior of coalitions of traders and middlemen is analyzed by an examination of *strong equilibria* as introduced by Aumann (1964). Strong equilibria are distributions of middlemen and traders from which no coalitions have a tendency to deviate. The collection of strong equilibria can be as seen as analogous to the Semi-Core in Gilles (1990), which gives those allocations that cannot be improved upon by any primitive coalition. In the market formation model, an allocation is then replaced by a distribution of middlemen and traders (the 'organizational

[2]Effectuation of these links could be stochastic (see, e.g., Kirman, Oddou and Weber (1986), or deterministic (see, e.g., Kalai, Postlewaite and Roberts (1978), and Borm, Owen and Tijs (1992))

structure' of the economy). One is also referred to Gilles (1996).

The market formation model incorporates coalitions of middlemen as necessary entities to perform transactions; direct interactions between traders are not possible. Efficiency is enhanced by the presence of these middlemen, since gains-from-trade can be extracted for the traders. At the same time, middlemen take advantage of their position by collecting a fee. An experimental study by Williams and Smith (1984) shows that middlemen are indeed capable of taking payoffs while enabling trade. There are situations though in which middlemen do not take advantage of their position, as is shown by Kalai, Postlewaite and Roberts (1978) for middlemen in trade economies with limited communication. Middlemen may be distinguished into *market makers* and *match makers*. Match makers are middlemen who bring market parties together; They are not concerned with the activities of the agents after they have been matched. In this way, a matchmaker is opposite to a *market maker*, who matches different types only indirectly, for instance by buying a commodity and reselling it. Yavas (1993) compares the performance of market makers and match makers in different kind of markets. Both the market formation model and the bilateral matching model incorporate match makers.

Rubinstein and Wolinsky (1987), and Bhattacharya and Yavas (1993) analyze models of middlemen in commodity markets. In their models, bilateral transactions between sellers and buyers, sellers and middlemen, or buyers and middlemen are considered. In the model of Rubinstein and Wolinsky, sellers, buyers and middlemen are randomly matched with exogenous probabilities, to perform transactions. In the model of Bhattacharya and Yavas, one middleman out of the set of potential middlemen is designated by a central exchange authority, whereafter trading agents decide whether or not to trade via the middleman. In both models, middlemen buy units of the commodity from sellers, and sell them to buyers. In Bhattacharya and Yavas, middlemen become active because sellers and buyers have to search for one another, which is costly; Middlemen are able to decrease search costs.

Middlemen in the market formation model activate markets to the extent that they make it possible for traders to exchange commodities at competitive prices; they are not involved in the exchange process. Hence, although the market formation model may say something about the emergence of 'market places', it does not provide insight into the market process, specifically, how competitive prices are formed; this remains a black box. The problem of price formation processes in general equilibrium models was recognized already by Walras (1874), who introduced a tatonnement process to describe price formation. Alternative price formation algorithms have been developed by, among others, van der Laan and Talman (1987). Spanjers (1992) develops models of hierarchies where commodity prices are set by higher ranking agents for lower ranking agents.

Shapley and Shubik (1977) develop a strategic market game to explain the setting of commodity prices. Consumers follow a posted offer/posted bid procedure. Every consumer posts his offer and bid for each commodity at the *trading post* corresponding to that commodity. At the trading post for commodity k, every consumer posts his offer and bid for commodity k. In the market formation model, commodity markets are local in the sense that no commodoties are traded between markets. These local markets could be interpreted as trading posts. The difference with Shapley and Shubik, is that at each local market, *all* commodities are traded by a *subset* of agents, whereas at a trading post, *one* commodity is traded by *all* agents. In their strategic market game, trading posts are characterized by commodities. In the market formation game, local markets are characterized by access fees and the composition of supply. The strategic market game approach is followed further in, for example, Dubey (1982), and Nti and Shubik (1984). The posted offer/posted bid procedure has been tested experimentally by Williams (1973), and Davis and Williams (1986).

Littlechild and Owen (1980) develop a model of price formation in the spirit of Kirzner (1973,79). Kirzner argues that certain agents recognize opportunities for profitable innovations in an economy, and thenceforth operate as 'entrepreneurs' in order to implement these innovations.[3] Littlechild and Owen (1980) introduce entrepreneurs in the form of traders, who recognize arbitrage opportunities between different markets and, subsequently, buy and sell on profitable markets. In equilibrium, different markets may have different commodity prices. This is analogous to the market formation model. In Littlechild and Owen's model, commodity prices form a signal for traders to become active. In the market formation model, commodity prices give an indication for which types it is profitable to become middlemen or traders. Since prices determine terms-of-trade, consumers will tend to become middlemen at markets that have unfavorable terms-of-trade, and to become traders otherwise.

[3]This phenomenon was corroborated experimentally by Forsythe, Nelson, Neumann and Wright (1993).

1.2 Public aspects of market organization

The markets in the market formation model, and the matching institution in the bilateral matching model may be considered as public goods, to the extent that the use of a market or a match maker's service by one agent does not prevent other agents from using it. These public goods are costly; endowments of commodities or money must be sacrificed in order to provide them. Gilles, Diamantaras and Ruys (1996) incorporate the organization of a Walrasian economy into a public project, called *trade infrastructure.* The output of a trade infrastructure is a market that improves the terms of trade for its members. Such a public project can be incorporated as a private enterprise owned by the middlemen. The cost of an infrastructure consist of a set-up cost, being the cost of building the project, an access cost, being the cost of consumers entering the market supported by the infrastructure, and a transaction cost, being the costs of market transactions.

In a general equilibrium framework, Gilles et al. derive valuation equilibria. A valuation equilibrium is a tuple of commodity prices, inputs for a particular public project, and consumptions of private commodities and the public project such that every agent maximizes his utility, demand equals supply, and budgets are balanced.[4] The market formation model may be seen as a positive implementation of their normative model. The set of markets is then analogous to the collection of trade infrastructures, the set-up costs of a project to the entrance cost of a market for middlemen, and the access cost of a project to the access fee of a market for consumers. A market equilibrium with one active market is analogous to a valuation equilibrium. Notice that the market formation game can have equilibria with several active markets, which has no analogy in Gilles et al., they having only one global infrastructure in equilibrium.

In the literature on *club economies*[5], private goods are traded locally in the presence of (local) public goods. In the model of Gilles and Scotchmer (1997), for instance, members of the same club share a public good, and trade private commodities amongst each other. An admission price is paid from private commodities to enter a club. The agents face externalities from the presence of other agents. The club economy model is analogous to the trade infrastructure model of Gilles, Diamantaras and Ruys (1996) to the extent

[4]See also, e.g., Chatterjee and Corbae (1994).

[5]The club concept was introduced by Tiebout (1956), and Buchanan (1965). Sandler and Tschirhart (1980) provide a survey of club theory in economics.

that both models have trade facilitated by public projects. The difference lies in the locality of trade; the public project of Gilles, Diamantaras and Ruys (1996) supports a global competitive market, whereas in Gilles and Scotchmer (1997) local markets are considered.

We may liken the market formation model to the club economy model of Gilles and Scotchmer in the following ways. First, an active market in the market formation model is analogous to a local public good in the club economy model. An active market is public in the sense that traders use it simultaneously and non-exclusively; It is local in the sense that traders at other markets do not use it. Olson (1965) distinguishes between inclusive and exclusive clubs. Inclusive clubs share pure public goods and require no restrictions on the size of membership, whereas exclusive clubs share impure public goods and require a size limitation owing to crowding. Secondly, the access fee in the market formation model is analogous to the admission price of a club in the club economy model. Both the access fee and the admission price are paid from initial endowments before entering respectively a market and a club. Thirdly, both models have non-anonymous crowding effects because the composition of trader types at a market or club influences one's utility. Scotchmer and Wooders (1987) compare anonymous and non-anonymous crowding. Under anonymous crowding, only the number of agents matters, not their types. In the market formation model, consumers face non-anymous crowding, whereas crowding for middleman is anonymous.

1.3 Evolutionary selection mechanisms

In Chapter 4, the formation of markets is modelled as an *evolutionary* process. Evolutionary game theory was originally developed in theoretical biology: instead of societies of economic agents, populations of animals were considered. In these populations, animals meet randomly, whereupon they take certain actions. Animals that take actions that are 'successful' relative to other actions, given the distribution of actions over the populations, are 'rewarded' by having more offspring than others. It is then assumed that the offspring of an animal inherits the behavior of its parent; Therefore the 'successful' behavior becomes more prominent in a next generation, while less 'successful' behavior tends to extinction. In an economic context, animals are replaced by boundedly rational agents. The mechanism of behavior inherited by offspring is replaced by the mechanism of social imitation: 'more successful' actions are imitated by more agents the following 'generation'. An overview of biological models is offered by, for example, Maynard Smith (1982), and Van Damme (1991). Van Damme (1994), and Weibull (1995) give overviews of recent developments in evolutionary game theory.

A specific model of this kind of behavioral evolution was developed by Taylor and Jonker (1978). They consider large populations in which agents/animals are randomly matched in pairs, after which they play a one-stage matrix game. The distribution of actions over the population is assumed to evolve in continuous time, where the growth rate of each fraction (of agents having adopted a certain action) is equal to the difference between the payoff obtained by taking that action, given the distribution over all actions, and the weighted average of all payoffs in the population. This means that actions yielding more than the average payoff are played more frequently in the next period/generation. Under this mechanism of social imitation (the *replicator dynamic*), Taylor and Jonker show that the distribution of behavior converges towards a Nash-equilibrium of the game, that is, a distribution in which no agent has an incentive to adopt a different action, given the actions taken by all other agents. The replicator dynamic thus can be seen explanating Nash-equilibrium, the most commonly used equilibrium concept in standard game-theory, without assuming perfect rationality.

The replicator dynamic is an example of an *evolutionary dynamic*. Friedman (1991), and Samuelson and Zhang (1992) analyze more general models of evolutionary dynamics. They find, for quite general dynamics, that distributions of behavior tend to Nash equilibria. These results show, that specific knowledge about the form of an evolutionary dynamic is not so important in making predictions about long-term patterns of behavior. We also refer to Nachbar (1990), for an analysis of evolutionary dynamics in discrete-time models.

An important feature of evolutionary dynamics is that they are *deterministic*. Stochastic considerations are made, however, to interprete the long-term (stable) outcomes of the dynamic. Distributions of actions are stable, if sufficiently small random perturbations of the distribution tend to 'die out'. This means that if small groups of agents change to different strategies, although they have no incentive to do this while in a stable distribution, they will play their original actions in the long run.

Although stochastic perturbations of distributions are considered in the theory of deterministic dynamics, these perturbations are a one-time event and exogenous; they are not part of the dynamic itself. Kandori, Mailath and Rob (1993), Young (1993), Fudenberg and Harris (1992), Young and Foster (1991), and Hurkens (1994) analyze models in which stochastic perturbations are an integral part of the evolutionary mechanism. Their models have two components. First, a *learning* component: agents learn to play 'successful' actions based on their experiences with past plays of the game. Second, an *experimentation* component: occasionally, agents do not play actions which are 'optimal' given past experiences, but make 'mistakes'. The possibility of mistakes makes that distributions of actions never 'settle down', but continuously are subject to 'drift'. The models of Kandori et al. (1993) and Fudenberg and Harris (1992) consider stochastic versions of evolutionary dynamics, whereas Young (1993), and Hurkens (1994) model evolution by a process of *adaptive learning.* Adaptive learning differs from evolutionary dynamics in the sense that the decision making of the agents is modelled explicitly by adaptive learning; agents take 'optimal' actions given the games played in the past. Again, Nash equilibria appear to be the long-term distributions under certain conditions.

1.4 Overview of the monograph

In the Chapters 2, 3 and 4 we analyze models of market formation and selection by middlemen and traders, which markets perform as competitive markets. In Chapter 2, we develop and analyze a *market formation game* which is played by Arrow/Debreu type consumers in a large economy with a finite number of economic types. An economic type is characterized by an endowment bundle and a utility function. A population of consumers together with their individual characteristics is the economic *primitive* of the model. The institutional environment is given by a countable collection Γ of local, costly competitive markets.

The market formation game is a one-shot game in which all consumers choose a market out of the collection Γ, and a *role*, namely *middleman* or *trader*. Positively measured coalitions of middlemen *activate* markets, that is, they make it possible for the traders at that market to trade their endowment bundles for competitive equilibrium bundles. Markets have the following characteristics.

1. Markets are *costly.*

2. Trade is *local.*

3. There is no recontracting.

Locality of trade and no recontracting are the institutional characteristics of markets; they specify the rules of trade.

Ad 1

The costs of markets are incurred by both middlemen and traders. Middlemen incur two kinds of cost. First, an *opportunity cost* results from the organizational effort associated with market activities. The opportunity cost takes the form of foregone gains-from-trade, that is, middlemen are not allowed to trade their endowments. Secondly, a *set-up cost* in the form of a market-specific reduction of endowments. Whereas the opportunity cost results from performing the middleman's job, the set-up cost reflects the investment needed to reach the position of middleman. The set-up cost may be physical or non-physical. Physical entrance costs are incurred by the building of market facilities. A traditional open market provides and example: market salesmen are middlemen that have invested in stands and have bought sales permits. Another possibility is that an entrance cost reflects a loss of facilities for producing endowments. An example is the already described agricultural economy, where a farmer can become a middleman by providing other farmers with facilities to trade crops. To do this, he has to give up crop growing; the cost of the market he sets up is then his entire endowment (his crop). An example of a non-physical entrance cost is learning: in order to organize a market, a middleman has to acquire the customs and techniques of that market. The stock market is an illustration of this.

Traders' costs are given by an *access fee*, uniform for all consumers, which is the cost of using the market. On entering a market as trader, a consumer's endowment is reduced by the access fee, whereafter he trades his resulting net endowment. The total bundle collected from all traders' access fees in a particular market is transferred to the middlemen of that market, in such a way that every middleman receives an equal bundle. This makes sense if we assume that middlemen operate independently, and, with the absence of differentiation in access fees among middlemen, traders disperse themselves equally over them; this yields to every middleman an equal share of consumer types, and hence an identical access fee.

Ad 2

Locality of trade is an institutional characteristic of markets. Commodity trade is local in the sense that trading of the net consumers' endowment bundles at a certain market depends entirely on the consumer types at that market. No bundles are traded between markets. A plausible reason could be that transport between markets is (very) costly.

Ad 3

The characteristic of no recontracting means that after traders have left their selected market, they cannot retrade the bundles they have exchanged. If there were one global competitive market, consumers would not have an incentive for recontracting, since their traded bundles resulting from endowments reduced by access fees would be efficient given the cost structure. With several local markets, however, this incentive may clearly exist: although traded bundles are efficient *given* locality of trade, they need not be *first best* even when institutional restrictions determine feasibility. No recontracting seems reasonable, however, if one considers it as a time and effort constraint on finding trading partners outside the market.

The payoffs of the market formation game are the (expected) utility levels of the commodity bundles with which consumers leave the market, given the distribution of middlemen and traders over markets. For a trader, this is the local competitive equilibrium bundle traded for his net endowment bundle. For a middleman, this is a share of the total collected bundle which is raised from the access fees of all traders trading at his market. An agent's choice of market and role is based upon *a priori* expectations about his resulting commodity bundles.

The market formation game describes the formation of local, costly markets, rather than one global, costless, market. The individual rational decisions of agents generate an aggregate Nash equilibrium distribution of agents over markets and roles as an outcome of the game[6]. In a Nash equilibrium, no consumer has an incentive to select a different role or market, given the choices of all other consumers. A subset of Nash-equilibria is formed by the *no-trade* equilibria in which no market is active, that is, has a positive fraction of middlemen. An active market is necessary for trade to take place at all[7]. In a no-trade equilibrium, all traders consume their endowments. Of greater interest is the set of *market equilibria.* In a market equilibrium, at least one market is active, every active market is used by at least two positive fractions of traders, and all traders are at active markets. Hence, in a market equilibrium all traders extract gains-from-trade, provided access fees are not too large.

A necessary condition for market equilibrium is that the payoffs of consumers of the same type are equal if they are members of positive fractions. This condition reflects free entry to roles and markets, in the sense that each agent may become a middleman or trader at any market. As long as, at a certain market, one role yields higher payoffs than the other, consumers will enter the market adopting the more profitable role. We could say that there exists a 'competive market' for the setting up of commodity markets; the distribution of consumers over markets is competitive in the sense that, in equilibrium, there is nothing to gain from choosing one role over another. Notice that consumers enter markets independently; if, though, middlemen colluded for instance, they would be able to exert market power, thereby preventing other middlemen entering[8].

In making decisions, consumers have to take into account the costs of markets on the one hand, and the composition of consumer types already at the markets on the other. If a consumer becomes a middleman, his payoff is reduced in proportion to the number of other middlemen at his market. If he becomes a trader, his gains-from-trade depend heavily on the other trader types at his market. Because all agents make their decisions simultaneously, this market 'externality' poses a considerable coordination problem for consumers. In the absence of such an externality, only the costs of markets would matter: for instance, if consumers buy a single homogeneous commodity from one of several firms,

[6] Krugman (1991,1993) analyzes macroeconomic models in which agents disperse themselves over different markets. His models, however, lack a microeconomic foundation. Krugman models the aggregate behavior of agents exogenously, whereas we derive aggregate from individual behavior.

[7] A zero middleman fraction implies that each middleman would have to serve an infinite number of traders.

[8] Greif, Milgrom and Weingast (1994) consider collusion between 'trade centers' in medieval trade organizations, such as the Hanseatic League, which consisted of a large number of cities, each 'monitoring' the others in their treatment of visiting traders.

the Nash equilibrium is that all consumers buy from the cheapest firm. The coordination problem may cause all kinds of Pareto-inferior lock-ins, for instance by all consumers coordinating on a market with a high access fee. Since consumers are not able to activate a market individually, they do not necessarily gain by switching to a market with a lower access fee.

The inability of zero-measured coalitions to activate and use markets profitably is an *intertia* of the market formation model, because any arbitrarily small positively sized coalition containing middlemen and traders of different types is able to activate and use a market profitably. Another inertia is the inability of traders being members of zero-measured coalitions to trade, caused by assuming a continuum of consumers. With a countable number of agents, individual consumers would be able to trade at active markets. The first-mentioned inertia, however, also applies to economies with a countable number of agents; to activate and use a market effectively, at least three agents are needed: one middleman and two consumers of different types.

The second inertia generates a large number of Nash equilibria. We show that an arbitrary collection of markets with sufficiently small access fees may be activated in equilibrium, provided the number of types is large enough. In an equilibrium, any distribution of traders over active markets with all traders of one type at the same market is feasible, in principle. The inertia is resolved if one takes coalitions as decisive units instead of individual consumers. Coalitional stability of equilibria is captured by the concept of *strong* equilibrium as introduced by Harsanyi (1973). A Nash equilibrium is strong if there does not exist a coalition yielding all its members strictly higher payoffs. A straightforward result is that an equilibrium with no active markets is not strong if there is a market with a sufficiently small access fee. By the heavy dependency of competitive equilibrium on the composition of trader types, strongness properties of market equilibria are less straightforward. To obtain some insights, we analyze strongness properties of market equilibria in economies with Cobb-Douglas utility functions. For markets with proportional access fees, we show that strong equilibria necessarily have only the market with the lowest access fee active.

The concept of strong equilibrium has a cooperative interpretation; it considers deviations by coalitions whose members coordinate actions amongst each other. If market formation would be modelled as a cooperative game, it would yield strong equilibria as

Core-elements. In a strong equilibrium, the assignment of consumers to roles and markets is such that no coalition is able to find an assignment that yields all its members a strict payoff improvement. In particular, the 'grand coalition' A cannot. Strong equilibria yield therefore an efficient division of payoffs, given the institutional structure of markets supporting the economy. Were roles and markets assigned by a social planner, strong equilibrium profiles would be candidate assignments.

In Chapter 3, a different version of the market formation game is analyzed. There, we have two consumer types with Cobb-Douglas utility functions, and two markets. The economy has a *spatial* character in the sense that consumer types and markets are associated with regions. There are two regions, each of which is inhabited by one consumer type. One market is located in each region. Consumers can become middleman in their own region only, as opposed to Chapter 2, where middleman have free access to all markets. The restriction that middlemen do not leave their own region, may have several interpretations. For instance, markets possess a high degree of region specificity, in the sense that learning the market customs and techniques requires too much effort for middlemen from the other region. Another interpretation is that it is legally not possible to become a middleman in the other region. This interpretation would make particularly sense if we think of the regions as countries. In that case, 'human capital' in terms of organizational skills is immobile. An extra cost is incurred by traders in the form of transportation costs, when they travel across regions. Transportation costs are measured in terms of some non-tradeable commodity.

Three types of Nash equilibria are found, namely, no-trade equilibria, market equilibria with *concentrated trade*, and market equilibria with *dispersed trade*. We have concentrated trade if one market is active and both consumer types are represented with a positive fraction there. We have dispersed trade if both markets are active and have both consumer types represented by positive fractions. Like in Chapter 2, the strongness criterion is applied to these equilibria. Two types of strong equilibria are found. First, the equilibrium with concentrated trade at the market for which the combination of access fee and transportation costs is most favorable. That is, for which the access fee is relatively small, the cost from travelling to the other region is relatively large, and the cost from travelling from the other region is relatively small. If conditions are insufficiently favorable, a coalition of middlemen and traders can be found that is able to activate profitably the market in the other region. In this coalition, consumers inhabiting the region with the active market obtain higher gains-from-trade at the newly activated market to compensate for incurred transportation costs, whereas consumers from the other region sacrifice gains-from-trade in favor of them. This sacrifice is compensated by not having to bear transportation costs. Second, the equilibrium with dispersed trade is strong.

In Chapter 4, coalitional stability is considered in the context of *boundedly rational* agents. Whereas the strongness condition in Chapters 2 and 3 increases the degree of rationality of players beyond the level leading to a Nash equilibrium, in Chapter 4 rationality is decreased. There, an *evolutionary stability* concept is applied to the market formation game. The idea behind evolutionary stability is that the one-shot market formation game is played over and over again, and in every period, players play actions with relatively large payoffs with a higher frequency in a next period. Hence, rather than calculating perfectly the effects of their actions on others and take actions accordingly, players now 'imitate' currently profitable actions. If the state space is differentiable, the *replicator dynamic* yields evolutionary stable distributions in the long run.

Chapter 4 is two-fold. In the first part, we derive evolutionary stability properties for the market formation model of Chapter 2. The second part considers dynamic properties under the replicator dynamic of a model similar to that of Chapter 3. In Chapter 2, the existence of two kinds of market equilibria is derived, in which several markets with sufficiently small access fees are activated. The first kind applies to a situation where markets have relatively small set-up costs, the second kind where set-up costs are not necessarily small. We apply the static evolutionary stability concept to these equilibria. It is shown that equilibria of the first kind are evolutionary stable, whereas equilibria of the second kind are evolutionary stable if the access fee of at least one active market is sufficiently small.

Evolutionary stability is a sufficient condition for asymptotic stability under the replicator dynamic if payoff functions are differentiable on the state space. In the market formation model of Chapter 2, this is not the case; a payoff discontinuity for zero fractions of middlemen and consumers exists. We propose a 'smoothing' procedure by assuming that 'negligible', i.e., positive but very small, fractions obtain nearly the same payoffs as zero fractions. With respect to middlemen, we may argue that small fractions are not able to activate a market properly; the small group of middleman causes considerable inefficiencies. With respect to consumers, we could have that negligible groups are not 'recognized' as trade partners.

If the state space is not 'smoothened' on the boundaries, evolutionary stability does no longer guarantee asymptotic stability. In the second part of this chapter, we analyze asymptotic stability of equilibria if the state space is discontinuous on the boundaries. We analyze a model with two types and two markets, similar to the model of Chapter 3, to which the replicator dynamic is applied. This model yields two kinds of asymptotically stable states. In the first kind, all consumers select the market, either as a middleman or a trader, for which the combination of access fee and transportation costs is most favorable, basically the same as the condition for strongness of the equivalent market equilibrium in Chapter 3. In fact, we show that strongness and evolutionary stability coincide. Further, the equilibrium with dispersed trade is asymptotically stable.

In Chapter 5, we study bilateral matching with costly intermediation. With bilateral matching, there are two types of agents, each agent seeking to be matched with an agent of the other type, to exchange commodities or services. The real-estate market is an example. There, buyers typically seek one seller of a house. Another example is the marriage market; men desire to be matched with women one-to-one and vice versa. As a third example, we mention companies searching for entertainers to perform at special occasions.

It may be difficult or impossible for market participants to establish matches on their own account. Problems are incurred mainly in obtaining information about desirable matching partners. Special skills may be required for extracting the right information, as found with intermediating agencies, such as real estate brokers, dating agencies and theater agencies. Brokers are examples of *match makers*, the intermediating institutions in this chapter. Match makers bring agents of both market sides together, without being involved in the interaction between them. In the housing market, for instance, brokers rather than buying houses and reselling them again, provide sellers the opportunity to sell houses themselves.

We study a model with two market parties (buyers and sellers), distributed uniformly on a circle, with possibly different densities. Each buyer desires to be matched with a specific seller, to be specific, with the seller who is located at the same location on the circle. Hence, there is a maximal degree of differentiation between buyers; there is exactly one type of seller making a profitable exchange possible. Buyers and sellers are matched by two middlemen (match makers), located symmetrically on the circle, that is, diametrically opposite to each other.

Every agent going to a middleman pays him a commission fee, and incurs a certain relational cost, provided he is matched. Relational costs are proportional to the distance along the circle to a middleman[9]. Furthermore, each agent has a reservation price, indicating how much he is willing to spend, in terms of fee plus relational cost, in order to be matched. The reservation price may differ over types.

With respect to buyers and sellers, we make the assumption that each of them goes to the middleman whose sum of fee and relational cost is the smaller. Also, the middlemen expect the agents to behave that way. We do not incorporate more sophisticated expectations of middlemen with respect to agents' behavior, or of the agents with respect to the other agents. If they would do so, the risk of not being matched would enter the agents' utility function. We think there is no strong reason to incorporate such expectations, since neither the middlemen nor the agents possess any *a priori* information, based on which they could form any reasonable expectations about the other agents' behavior. Because we focus on the competition in commission fees by middlemen, we choose not to complicate the model unnecessarily. It should be noted, that the behavioral assumptions made are consistent with strategic behavior. Given that the middlemen set equilibrium fees, the buyers and sellers cannot do better than indeed go the 'cheapest' middleman.

The game of setting commission fees played by the middlemen is analyzed for two different cases. First, the case where densities of sellers and buyers are equal. Then, at every location on the circle, the number of buyers and sellers is equal. Second, the case where densities are unequal, specifically, if the density of sellers is smaller than the density of buyers, in which case a certain number of buyers remains unmatched at every location. The two cases yield structurally different sets of equilibria when both reservation prices are sufficiently large, in which case there is competition for those agents that are indifferent between going to either one of the middlemen. If reservation prices are sufficiently low, there is no such competition, so that the middlemen establish 'local monopolies' in equilibrium. Then, obviously, the analysis is identical for both cases. We show that, generically, there exists a unique Nash equilibrium for the game when densities are un-

[9]Therefore, the model can be seen as a variant of the Salop (1979) model of spatial competition, where transportation costs take the place of relational costs.

equal, whereas in case of equal densities, there need not exist a unique equilibrium when competition occurs. In equilibrium, fees are such that every middleman has an equal share of type buyers and sellers This turns out to be the reason for the indeterminacy in the equal density case. Then, namely, the middlemen compete for both types of agents. If densities are unequal, only competition for one type occurs, namely the type with the lowest density, i.e, the sellers. Sellers then entirely determine the fee for buyers; the indeterminacy is resolved.

Among the equilibria, we find equilibria in which one type is charged a zero fee. The match makers even desire to subsidize this type, that is, charge a negative fee. In the local monopoly case, this kind of equilibrium occurs if the difference in reservation prices is sufficiently large. In that case, the type with the lower reservation price is 'free-riding'. The free-riding phenomenon arises from the fact that the payoff of a middleman is determined by the minimum of his shares of buyers and sellers. To increase his payoff, he needs to increase his shares of both types. In order to achieve this, the type with the low reservation price should be charged a relatively low fee, or even a zero fee. In that case, the 'low' type is needed only to attract the 'high' type, from which positive fees are collected.

The free-riding phenomenon is also found outside the local monopoly region. For the case with unequal densities, equilibria may exist where the short side of the market is served for free, under competition. The reason is that the middlemen compete for an indifferent agent only on the short side. The fee for the agents on the long side is chosen to adjust the share of these agents to the share of short side agents.

Chapter 2

A Strategic Market Formation Game

2.1 Introduction

This chapter defines and analyzes a strategic market formation game. We introduce an *economy*, which consists of two elements, namely an *economic primitive* and an institutional environment. The economic primitive describes individual characteristics, apart from their social (institutional) environment. It is formed by a continuum of consumers, partitioned into *types*, together with their individual attributes in the form of endowments and utility functions. The economy of the traditional competitive markets model (Arrow and Debreu (1954)) is an economic primitive embedded in a costless competitive global market. In our model, the costless global market is replaced by a set of costly local markets. Explicit organizers of these local markets are introduced in the form of *middlemen*.

Middlemen are able to *activate* markets, which means that they make it possible for the other agents, the *traders*, to trade on a competitive market. If a consumer enters a market as middleman, he pays the *set-up cost* of that market. The set-up cost is a part of endowments, and represents an investment in market activities. Further, middlemen bear an *opportunity cost* in the sense that they are not able to trade their resulting endowment bundle on the market. As compensation for these costs, middlemen obtain an *access fee* from the traders, which are paid from endowments. The traders' reduced endowments are then traded on the market[1]. Trade is strictly *local* in the sense that traded bundles at a certain market are depending entirely on the composition of trader types at that market.

Simultaneously, all consumers choose a market and a role (middleman or trader) unrestrictedly, such that their choices yield a Nash-equilibrium distribution over markets and roles. In a Nash equilibrium, no consumer has an incentive to select a different role or market, given the choices of all other consumers. A subset of Nash-equilibria is formed by the *no-trade* equilibria, in which no market is active, that is, has a positive fraction of middlemen. In a no-trade equilibrium, all traders consume their endowments. Of more interest is the set of *market equilibria.* In a market equilibrium, at least one market is active, every active market is used by at least two positive fractions of traders, and all traders are at active markets. Hence, in a market equilibrium all traders extract gains-from-trade.

We derive the existence of market equilibria for the cases of relatively small and large set-up costs. If access fees are sufficiently small for a subset of markets, numerous market equilibria with the markets in that subset being active are found. This is caused by the inertia associated with a continuum of traders, namely the property that traders being members of zero fractions are not able to trade. The derived equilibria have the property that all traders of the same type go to the same market. Therefore, no trader has an incentive to deviate to a different active market, as he is not able to trade there. This generates a large number of equilibria, in principle. Several of these equilibria could disappear if we would consider an economy with a countable set of consumers. Then, existence would become hard to proof, however, even in principle.

There is a second kind of inertia, namely the inability of zero sized coalitions to activate and use profitably a non-active market. This inertia may cause Pareto-inferior lock-in's of consumers into markets. For instance, consider a market equilibrium where all consumers select a market with a high access fee. It could be a Pareto-improvement if all consumers select a market with a lower access fee instead. This kind of inertia cannot be solved by a countable set of consumers. Namely, to activate and use a market profitably, at least three consumers are needed: one middleman and two traders of different types.

[1]The cost structure of markets is similar to that of the *trade infrastructure* introduced by Gilles, Diamantaras and Ruys (1996). A trade infrastructure is a costly institution supporting a global competitive market.

Although individual consumers do not have an incentive to switch markets in equilibrium, *coalitions* of positive measure might have this incentive. We examine coalitional stability of Nash equilibria by applying the *strongness* criterion, introduced by Aumann (1960). An equilibrium is strong if there does not exist a coalition all of whose members strictly increase their payoffs by deviating. In particular, the 'grand coalition' consisting of all consumers cannot. Strong equilibria yield therefore an efficient division of payoffs, given the institutional structure of markets supporting the economy. Were roles and markets assigned by a social planner, strong equilibrium profiles would be candidate assignments.

A straightforward result is that a no-trade equilibrium is not strong if there exist markets with sufficiently small access fees. Then, coalitions of middlemen and traders exist that have an incentive to activate and use these markets. The strongness properties of market equilibria are less straightforward. The reason is that the reaction to changes in access fees of Walrasian equilibria is depending heavily on traders' endowments and utility functions. It is not true in general, for instance, that deviation to a market with a smaller access fee yields traders a higher payoff. It could well happen that gains-from-trade decrease, though net endowments become larger. Not being able to derive more general results, we illustrate the strongness properties of economies with Cobb-Douglas utility functions. We derive an example in which both kinds of inertia, leading to a large number of market equilibria, are solved.

Having mentioned countability of the set of consumers, and coalitional stability requirements as resolutions to the intertia of the market formation game, we mention a third possible resolution, namely making the game a sequential move game instead of a simultaneous move game. Sequential moving would resolve the considerable coordination problem consumers face under simultaneous moving. We analyze an example of a sequential move market formation game with four consumers, taking subgame perfectness as solution concept. A characteristic result is that the order of play matters considerably for the equilibrium outcomes.

This chapter is organized as follows. Section 2 introduces primitive economies, markets and roles, and formulates the market formation game. Section 3 examines the Nash equilibria of the market formation game. Two existence theorems for market equilibria are derived. Section 4 considers strongness properties of equilibria. Section 5 gives an example of a sequential move market formation game with four players. Finally, proofs are gathered in Section 6.

2.2 The Model

Let a measure space (A, Ξ, μ) be given, where A is a set of consumers, $\Xi \subset 2^A$ is a sigma-algebra on the set of consumers, and $\mu : \Xi \to [0,1]$ is a probability measure. The space A is partitioned into $n \in \mathbb{N}$, $n \geq 2$, subspaces $A^i, i \in N := \{1, \ldots, n\}$, with $\mu(A^i) = 1/n$ for every $i \in N$. A consumer from the i^{th} subset is referred to as a *type* i consumer. The assumption that each type has the same measure has been made for convenience only.

The consumers of a certain type are identical with respect to their individual economic characteristics, namely endowments and utility functions. Every consumer $a \in A$ may consume bundles out of a consumption space $X := \mathbb{R}^l_+$, where $l \geq 2$ is the finite number of commodities. We denote $L := \{1, \ldots, l\}$. The consumers of the same type may differ with respect to their *social* characteristics, namely their choices of markets and roles, as we discuss later on.

The distribution of endowments and utility functions in the population is described by the tuple $(w^i, u^i)_{i \in N}$, where $w^i \in X$ is the commodity endowment of a consumer of type $i \in N$, and $u^i : X \to \mathbb{R}$ is his utility function. The consumers together with their individual characteristics form an economic primitive.

Definition 2.2.1 *An* **economic primitive** *is a tuple* $E_0 = \langle (A^i, u^i, w^i)_{i \in N} \rangle$ *satisfying the following conditions.*

1. *For every* $i \in N$, $u^i : X \to \mathbb{R}$ *is strictly monotone, continuous, and strictly quasi-concave.*

2. $\sum_{i \in N} w^i_k > 0$ *for each* $k \in L$.

The collection of primitives is denoted by $\mathcal{E}_0$.

Under the above conditions on endowments and utility functions, gains-from-trade exist between different types. Trade may take place at a set of markets, indicated by a set Γ.

These markets are organized by some of the consumers, the *middlemen*. The other agents in the economy are *traders*, who use the markets. At a market, middlemen provide to traders the opportunity of performing commodity transactions.

To every market $\gamma \in \Gamma$ its *market characteristic* $\psi(\gamma) = (\sigma(\gamma), \phi(\gamma)) \in \Psi$ is assigned. Every market characteristic $\psi \in \Psi$ consists of a *set-up cost* $\sigma = \langle(\sigma_1^i, \ldots, \sigma_l^i)_{i \in N}\rangle \in X^n$, and an *access fee* $\phi = \langle(\phi_1^i, \ldots, \phi_l^i)_{i \in N}\rangle \in X^n$. Entrance costs and access fees are such that they are affordable for all types, that is, $\sigma_k^i \leq w_k^i$ and $\phi_k^i \leq w_k^i$ for all $i \in N$ and $k \in L$. The set-up cost $\sigma^i(\gamma)$ is paid at market $\gamma \in \Gamma$ by a middleman of type $i \in N$ from his endowment bundle. The access fee is the cost to a market of a trader, and is paid by him from his endowment bundle. Given that a type i consumer goes to market $\gamma \in \Gamma$ as a trader, his *net* endowment bundle $w^i - \phi^i(\gamma) \in X$ is traded at γ.

For technical reasons explained later on, we make the following assumption on the set of markets Γ.

Assumption 2.2.2 *There exists a non-empty subset of markets $\Gamma_0 \subset \Gamma$ with every market $\gamma_0 \in \Gamma_0$ satisfying $\phi_{0k}^i(\gamma_0) = 0$ for all $i \in N, k \in L$.*

The role of middleman and trader at a certain market is a strategic choice. We make a measurability assumption on the groups of middlemen and traders generated by those choices.

Assumption 2.2.3 *The choices of roles and markets by consumers partition the set A into a tuple $(A_{\gamma m}^i, A_{\gamma c}^i)_{i \in N, \gamma \in \Gamma}$, where every group $A_{\gamma r}^i \subset A^i$ of consumers of type $i \in N$ at market $\gamma \in \Gamma$ with role $r \in \{m, c\}$ is measurable.*

An essential institutional feature of the markets in Γ is that they are chosen on a *global* level, i.e., all consumers have free access to all markets, but perform on a *local* level, that is, trade activities taking place at market γ depend entirely on the traders going to γ; they are independent of the activities at all markets $\gamma' \neq \gamma$, and no trade between markets takes place. Another institutional feature is that after traders have left the market they used for transactions, there is *no recontracting*: the bundles that traders leave with are consumed, and cannot be traded again.

An economic primitive together with a collection of markets is referred to as an economy.

Definition 2.2.4 *An* **Economy** *is a tuple* $E = (E_0, \Gamma)$ *of an economic primitive* $E_0 \in \mathcal{E}_0$ *and a collection of markets* Γ.

The collection of economies is denoted by $\mathcal{E}$.

As mentioned earlier, consumers of the same type may differ *socially*, although they are identical with respect to their individual characteristics. The social characteristic of a consumer is given by the market he enters and by the role he plays, namely middleman or trader. A distribution of middlemen and traders over markets is generated endogenously by a one-stage game, the market formation game. Simultaneously, all consumers choose a market out of Γ, and, on their chosen market, the role of either middleman or trader. Thereby, every consumer has the same action set $\Sigma = \Gamma \times \{m, c\}$, m denoting the role of middleman, and c the role of trader. By symmetry between consumers and by measurability assumption 2.2.3, we are allowed to represent the distribution of consumers over markets and roles by means of a tuple $s = (s^1, s^2, \dots, s^n)$, with for every $i \in N$

$$s^i = (s^i_{\gamma m}, s^i_{\gamma c})_{\gamma \in \Gamma}.$$

Here, $s^i_{\gamma m} \geq 0$ is the fraction of middlemen of type $i \in N$ at market $\gamma \in \Gamma$, and $s^i_{\gamma c} \geq 0$ the fraction of traders of type i at γ. We have for every $i \in N$

$$\sum_{\gamma \in \Gamma} (s^i_{\gamma m} + s^i_{\gamma c}) = 1.$$

That is, all consumers choose a market and a role, and their choices are exclusive. The tuples s are referred to as *states*. The collection of all states is denoted by Δ.

A market enables traders to perform transactions if there are middlemen at that market. To make this statement precise, we say a market $\gamma \in \Gamma$ is **active** at $s \in \Delta$ if there is a strictly positive fraction of middlemen at that market: $\sum_{i \in N} s^i_{\gamma m} > 0$. Otherwise, market γ is non-active at s.

On entering market $\gamma \in \Gamma$ with characteristic $\psi(\gamma) = (\sigma(\gamma), \phi(\gamma))$ as a trader, a type $i \in N$ consumer pays the access fee $\phi^i(\gamma)$ to the middlemen at γ from his endowment, provided γ is active. This leaves him with a net endowment

$$\widetilde{w}^i_\gamma(s) = w^i - \phi^i(\gamma).$$

His net endowment bundle is then traded for a competitive equilibrium bundle, which is determined by the composition of trader types at market γ.

Given a price vector $p \in \mathcal{P} := \{p \in \mathbb{R}^l \mid p_k > 0$ for every $k \in L$ and $\sum_{k=1}^l p_k = 1\}$ and a reduced endowment bundle $\widetilde{w}^i_\gamma(s)$ at an active market $\gamma \in \Gamma$ for state $s \in \Delta$, a type $i \in N$ consumer has a demand function

$$d^i(p, \widetilde{w}^i_\gamma(s)) = \text{argmax}_{x \in X}\{u^i(x) \mid p \cdot x \leq p \cdot \widetilde{w}^i_\gamma(s)\}.$$

By the assumptions of strict quasiconcavity and continuity of the utility functions, demand functions are determined uniquely and are continuous on $\mathcal{P}$.

Given a state $s \in \Delta$ and an active market $\gamma \in \Gamma$, at γ bundles $x^i_\gamma(s)$, $i \in N$, are traded against a local price $p_\gamma(s) \in \mathcal{P}$ such that every type $i \in N$ trader whose fraction is positive ($s^i_{\gamma c} > 0$) receives his demanded bundle

$$x^i_\gamma(s) = d^i(p_\gamma(s), \widetilde{w}^i_\gamma(s)).$$

At market γ, the equality of aggregate demand and aggregate supply yields an equilibrium.

Definition 2.2.5 *An* **equilibrium** *at market $\gamma \in \Gamma$ is a tuple*

$$\langle x_\gamma(s), p_\gamma(s)\rangle = \langle (x^1_\gamma(s),\ x^2_\gamma(s), \dots, x^n_\gamma(s)),\ p_\gamma(s)\rangle \in X^n \times \mathcal{P}$$

of commodity bundles and prices such that

1. *If market γ is active then*
   (a) *If $s^i_{\gamma c} > 0$ for $i \in N$, then every type i trader obtains his demanded bundle: $x^i_\gamma(s) = d^i_\gamma(p_\gamma(s))$.*
   (b) *If $s^i_{\gamma c} = 0$ for $i \in N$, then every type i trader obtains his reduced endowment: $x^i_\gamma(s) := \widetilde{w}^i_\gamma(s)$.*

(c) *Aggregate supply equals aggregate demand (*market clearing*):*

$$z(p_\gamma(s)) := \sum_{i \in N} (x^i_\gamma(s) - \tilde{w}^i_\gamma(s)) s^i_{\gamma c} = 0 \in X.$$

2. *If market γ is not active, then every type $i \in N$ trader returns with his endowment:* $x^i_\gamma(s) := w^i$.

We introduce the correspondence

$$W : \mathcal{E} \times \Delta \mapsto \prod_{\gamma \in \Gamma} \left(2^{X^n \times \mathcal{P}}\right)$$

that assigns to every economy $E = (E_0, \Gamma) \in \mathcal{E}$ and state $s \in \Delta$ a tuple of sets of equilibria $W(E, s) = (W_\gamma(E, s))_{\gamma \in \Gamma} \subset \prod_{\gamma \in \Gamma} \left(2^{X^n \times \mathcal{P}}\right)$. Under the assumptions made on utility functions and endowments in the definition of a primitive economy, existence of non-empty equilibrium sets $W_\gamma(E, s)$ at active markets γ is derived from Theorem 8.3 in Dierker (1974, p. 78):

Lemma 2.2.6 *Let a state $s \in \Delta$ be such that $\gamma \in \Gamma$ is an active market. Then the equilibrium set $W_\gamma(E, s)$ is non-empty for every economy $E \in \mathcal{E}$.*

In order to derive existence of Nash-equilibria of the market formation game, we need continuity of the correspondence W on the space of economies $\mathcal{E}$. Namely, we use certain conditions on access fees and endowments to derive an equilibrium. Continuity is not straigthforwardly satisfied. Consider for instance an economy with two goods. Prices can be normalized, so that excess demand $z_1(p_1)$ of commodity 1 at some market is expressed as the price p_1 of commodity 1. Suppose excess demand is like in Figure 2.2. Then, the correspondence is not continuous; an arbitrarily small change in endowments yields an economy with either one, or three equilibria, whereas the original economy has two.

For a non-empty subset of markets $\Gamma^* \subset \Gamma$ define the subspace

$$\Delta_+(\Gamma^*) := \{s \in \Delta \mid \sum_{i \in N} s^i_{\gamma m} > 0 \text{ and } s^j_{\gamma c} > 0 \text{ for all } j \in N,\ \gamma \in \Gamma^*\}.$$

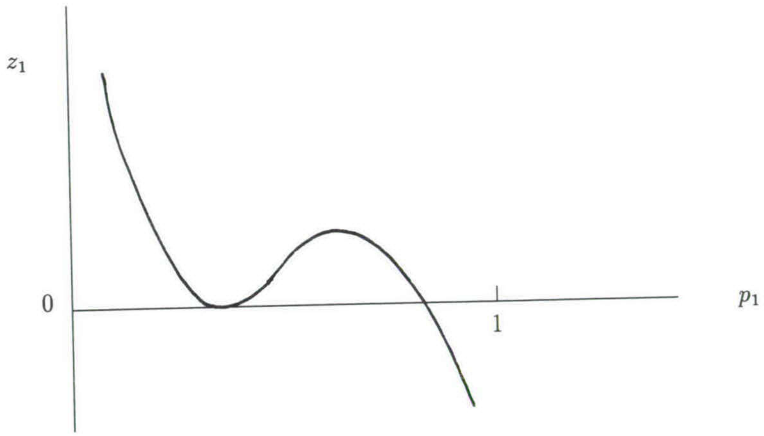

Figure 2.1: Excess demand function changing discontinuously with respect to $E \in \mathcal{E}$.

Though continuity is not satisfied in general, it is found for 'most' economies, as stated in the following lemma. Thereby, demand functions are endowed with the topology of uniform convergence.

Lemma 2.2.7 (cf. Corollary, page 85, Dierker (1974)). *For a non-empty subset* $\Gamma^* \subset \Gamma$, *let* $s \in \Delta_+(\Gamma^*)$. *Then at* s *the correspondence* $W_\gamma(\cdot, s)$ *is continuous on some residual subset* $\widetilde{\mathcal{E}}$ *of* $\mathcal{E}$. *That is, for each sequence* $\{E^k\}_{k\in\mathrm{N}}$ *converging to* $E \in \widetilde{\mathcal{E}}$ *the sequence* $\{W(E^k, s)\}_{k\in\mathrm{N}}$ *converges to* $W(E, s)$ *with respect to the Hausdorff distance.*

A *residual set* contains a countable intersection of dense and open sets. For example, among the reals, the collection of irrational numbers is a residual set, whereas the collection of rational numbers is not. A residual set is 'large' in a topological sense. For the special case that the equilibrium set is a singleton, continuity is satisfied on the entire collection $\mathcal{E}$.

Lemma 2.2.8 (cf. Proposition, page 82, Dierker (1974)) *For a non-empty subset* $\Gamma^* \subset \Gamma$, *let* $s \in \Delta_+(\Gamma^*)$. *For* $i \in N, \gamma \in \Gamma^*$: *if* $W_\gamma(E, s)$ *is a singleton at* $(E, s) \in \mathcal{E} \times \Delta$, *then* $W_\gamma(\cdot, s)$ *is continuous at* (E, s).

For the derivation of Nash equilibria, we need also continuity of the correspondence on the state space.

Lemma 2.2.9 *Let an economy $E \in \mathcal{E}$ be given. The correspondence $W(E,.)$ is continuous on the space $\Delta_+(\Gamma^*)$ for every $\Gamma^* \subset \Gamma$.*

PROOF

The demand functions $d^i_\gamma(.)$ are continuous on the price space $\mathcal{P}$. The demand equals supply equation implies that the set of equilibria responds continuously to changes in s.

$\square$

We assume further that each set of equilibria contains at least one equilibrium yielding some type a utility strictly higher than the utility of its initial endowment bundle at a market $\gamma_0 \in \Gamma_0$ having a zero access fee, which exists by Assumption 2.2.2. Although a competitive equilibrium outcome at γ_0 is always weakly individual rational, we need to assume strict individual rationality in the presence of costly institutions. Positive gains-from-trade may cover these costs.

Assumption 2.2.10 *Let be given a market $\gamma_0 \in \Gamma_0$, and a subset of types $M \subset N$ with $\#M \geq 2$.*

Let a state $s \in \Delta$ be such that

1. *γ_0 is active.*
2. *$s^i_{\gamma_0 c} > 0$ for every $i \in M$.*

Then for every $i \in M$ it holds that $u^i(x^i_{\gamma_0}(s)) > u^i(w^i)$ for some $(x_{\gamma_0}(s), p_{\gamma_0}(s)) \in W^i_{\gamma_0}(E,s)$.

The following example shows that Assumption 2.2.10 is not redundant.

Example 2.2.11 Suppose there are 3 consumer types. All types have Cobb-Douglas utility functions $u(x, y) = xy$ over two commodities. Type 1 consumers have endowments (1,1), type 2 consumers have endowments (3,1), and type 3 consumers have endowments (1,3).

Let market γ_0 be active, with a fraction $s^1_{\gamma_0 c} > 0$ of type 1 traders, and fractions 1 of type 2 and 3 traders. Then the market price of the second commodity, having normalized the price of the first commodity to one, is $p(s^1_{\gamma_0 c}) = \frac{4+s^1_{\gamma_0 c}}{4+s^1_{\gamma_0 c}} = 1$ for all $s^1_{\gamma_0 c}$. The equilibrium utility of the type 1 traders is therefore equal to one for all $s^1_{\gamma_0 c}$, equal to the utility of their initial endowments.

□

Equilibrium sets are not singletons, typically. This poses an indeterminacy problem for the agents. Therefore, we endow each type $i \in N$ consumer with a *belief* $(\mu^i_\gamma(E, s))_{\gamma \in \Gamma}$ on the collection $(W^i_\gamma(E, s))_{\gamma \in \Gamma}$, for every $E \in \mathcal{E}$ and $s \in \Delta$, where

1. $\mu^i_\gamma(E, s)(x, p) \in (0, 1)$ for every $(x, p) \in W^i_\gamma(E, s)$.
2. $\int_{W^i_\gamma(E,s)} \mu^i_\gamma(E, s)(x, p) \mathrm{d}(x, p) = 1$.

The beliefs have a Bayesian spirit, in the sense that their supports (the equilibrium sets) are considered as exogenous states-of-nature. This is consistent with the competitive markets model to the extent that traders take market prices as given. We make a continuity assumption with respect to beliefs.

Assumption 2.2.12 *Beliefs are continuous at every economy $E \in \widetilde{\mathcal{E}}$. That is, for every $i \in N, \gamma \in \Gamma$, and given $s \in \Delta$ we have that for each sequence $\{E^k\}_{k \in \mathbb{N}}$ converging to E the sequence $\{[\mu^i_\gamma(E^k, s)(x, p)]_{(x,p) \in W^i_\gamma(E^k,s)}\}_{k \in \mathbb{N}}$ converges to $[\mu^i_\gamma(E, s)(x, p)]_{(x,p) \in W^i_\gamma(E,s)}$ with respect to the Hausdorff distance.*

The *ex ante* payoff of a type i trader going to market γ, based on which he picks his action, is

$$f^i_{\gamma c}(s) = \int_{W^i_\gamma(E,s)} \mu^i_\gamma(E, s)(x, p) u^i(x) \mathrm{d}(x, p),$$

being the expected utility of the bundles in the equilibrium set, given his belief. In case the Walrasian equilibrium is unique, traders' *ex ante* expected payoffs are equal to *ex post* realized payoffs. If the equilibrium set contains multiple equilibria, expected payoffs are different from realized in general. We may well encounter the situation that certain traders leave a market with bundles that are not individually rational, though expected bundles are.

Lemma 2.2.9 and Assumption 2.2.12 yield

Lemma 2.2.13 *For* $i \in N$, $\gamma \in \Gamma$, $f^i_{\gamma c}$ *is continuous on the space* $\{s \in \Delta \mid \sum_{j \in N} s^j_{\gamma m} > 0$ *and* $s^i_{\gamma c} > 0\}$.

Under Assumption 2.2.10, payoffs at an active market with zero access fee $\gamma_0 \in \Gamma_0$ are strictly larger than the utility of initial endowments, provided at least two types of traders are represented by a positive fraction at γ_0:

Lemma 2.2.14 *Under the conditions of Assumption 2.2.10,* $f^i_{\gamma_0 c}(s) > u^i(w^i)$ *for every* $i \in N$.

Every middleman of type i leaves his market γ with his endowment reduced by the set-up cost, plus an equal share of the total collected access fee in case the market is active. For $s \in \Delta$, the middleman's bundle is the following.

- If γ is active, his bundle is $w^i - \sigma^i(\gamma) + \frac{\sum_{j \in N} \phi^j(\gamma) s^j_{\gamma c}}{\sum_{k \in N} s^k_{\gamma m}}$.
- If γ is not active, his bundle is $w^i - \sigma^i(\gamma)$.

The middleman's payoff is $f^i_{\gamma m}(s)$, being the utility of his bundle.

Now we are able to define the market formation game.

Definition 2.2.15 *The* **market formation game** *is a tuple* $G = (A^i, \Sigma, f^i)_{i \in N}$, *where*

- $A = \cup_{i \in N} A^i$ *is the set of players.*

- $\Sigma = \Gamma \times \{m, c\}$ *is the strategy set of every player* $a \in A$.
- *For every* $a \in A^i$*:* f^i *a tuple of payoff functions* $f^i_{\gamma r} : \Delta \to \mathbb{R}$, $(\gamma, r) \in \Sigma$, *for all* $i \in N$.

2.3 Equilibria of the market formation game

Next, we consider the Nash-equilibrium states of the market formation game. In a Nash-equilibrium state, no consumer has an incentive to take on a different role, or to switch to a different market, given the choices of all other consumers.

Definition 2.3.1 *A state* $s^* \in \Delta$ *is a* **Nash equilibrium** *if for all* $i \in N$ *and* $(\hat{\gamma}, \hat{r}) \in \Sigma$ *with* $s^{*i}_{\hat{\gamma}\hat{r}} > 0$ *it holds that*

$$f^i_{\gamma r}(s^*) \leq f^i_{\hat{\gamma}\hat{r}}(s^*) \text{ for every } (\gamma, r) \in \Sigma.$$

This definition is equivalent (see, e.g., Friedman (1991)), to saying that for every $i \in N$ and $(\gamma, r), (\gamma', r') \in \Sigma$

$$\begin{cases} f^i_{\gamma r}(s^*) = f^i_{\gamma' r'}(s^*) & \text{if } s^{*i}_{\gamma r}, s^{*i}_{\gamma' r'} > 0 \\ f^i_{\gamma r}(s^*) \leq f^i_{\gamma' r'}(s^*) & \text{if } s^{*i}_{\gamma r} = 0 \text{ and } s^{*i}_{\gamma' r'} > 0. \end{cases} \tag{2.1}$$

A first result is

Proposition 2.3.2 *Any state* $s \in \Delta$ *with every market* $\gamma \in \Gamma$ *non-active is a Nash equilibrium.*

Proof

We have $s^i_{\gamma m} = 0$ for every $i \in N$ and $\gamma \in \Gamma$. Then $f^i_{\gamma m}(s) = u^i(w^i - \sigma^i(\gamma))$ and $f^i_{\gamma c}(s) = u^i(w^i)$, hence, $f^i_{\gamma m}(s) \leq f^i_{\gamma c}(s)$ for all i and γ, implying that s is a Nash equilibrium by (2.1).

If no market is active, we have a *no-trade* equilibrium. No trader is enabled to trade at any market and have to consume their endowments. This is a situation of 'autarcy'.

The model does not incorporate the possibility of consumers 'staying home' and consume their endowments. However, if a consumer becomes trader at a non-active market, this is the same for him as 'staying home' in terms of payoffs. Hence, the presence of at least one non-active market ensures that consumers will not be worse off in equilibrium than consuming their endowments. This is stated in the following lemma.

Lemma 2.3.3 *Let $s^* \in \Delta$ be a Nash equilibrium with at least one non-active market $\hat{\gamma} \in \Gamma$. Then $f^i_{\gamma r}(s^*) \geq u^i(w^i)$ for all $i \in N, (\gamma, r) \in \Sigma$ with $s^{*i}_{\gamma r} > 0$.*

PROOF

In s^* we have for every $i \in N$: $f^i_{\hat{\gamma}c}(s^*) = u^i(w^i)$. According to (2.1), we therefore have $f^i_{\gamma r}(s^*) \geq u^i(w^i)$ for all $i \in N, (\gamma, r) \in \Sigma$ with $s^{*i}_{\gamma r} > 0$.

□

Straigthforward from this lemma we obtain

Lemma 2.3.4 *Let $s^* \in \Delta$ be a Nash equilibrium with at least one non-active market $\hat{\gamma} \in \Gamma$. Suppose a market $\gamma \in \Gamma \setminus \Gamma_0$ is active, then necessarily $s^j_{\gamma c} > 0$ and $s^k_{\gamma c} > 0$ for some $j, k \in N$ with $j \neq k$.*

In the sequel, we focus on equilibria satisfying the condition of the lemma. We call them market equilibria.

Definition 2.3.5 *A Nash equilibrium $s \in \Delta$ is a* **market equilibrium** *if the following conditions are satisfied.*

1. *There is a non-empty subset $\Gamma^* \in \Gamma$ of active markets.*

2. *At every active market* $\gamma \in \Gamma^*$ *there are at least two different types* $i, j \in N$, $i \neq j$, *such that* $s^i_{\gamma c} > 0$ *and* $s^j_{\gamma c} > 0$.

3. *For every non-active market* $\gamma \in \Gamma \setminus \Gamma^*$ *we have* $s^i_{\gamma c} = 0$ *for all* $i \in N$.

The first condition distinguishes market equilibria from no-trade equilibria as in Proposition 2.3.2. The second condition is satisfied whenever there is at least one non-active market, by Lemma 2.3.3. The third condition may seem restrictive; it does not allow traders to go to non-active markets, consuming their endowments. Would this condition be deleted, also market equilibria in which only part of the traders has positive gains-from-trade are considered. It could namely be the case that access fees are such that no market equilibrium exists if *all* types are required to be at an active market, since some types' maximally obtainable gains-from-trade are only very small. The analysis does not change drastically by imposing the third condition, however.

In the remainder of this section, we state and explain two theorems, in which existence of market equilibria with several active markets having sufficiently small access fees is derived. The theorems distinguish between the cases of relatively small and large set-up costs.

Theorem 2.3.6 *Let be given numbers* $\delta > 0$ *and* $\epsilon > 0$, *and a non-empty subset of markets* $\Gamma^* \subset \Gamma$ *with* $\#\Gamma^* \leq n/2$ *such that for every* $\gamma \in \Gamma^*$, *the market characteristic* $\psi(\gamma) = (\sigma, \phi)$ *satisfies*

1. $||\phi^i|| < \delta$ *for all* $i \in N$.

2. $||\sigma^i|| < \epsilon$ *for all* $i \in N$.

If δ *and* ϵ *are sufficiently small, then there generically exists a market equilibrium* $\hat{s} \in \Delta$ *such that all markets* $\gamma \in \Gamma^*$ *are active.*

For the proof, we refer to Section 2.6. There, an equilibrium is derived in which one type disperses over the active markets as middlemen, whereas the other types become traders there. We illustrate this type of equilibrium by means of the following example.

Example 2.3.7 All consumers of type 2 to n, n odd, have Cobb-Douglas utility functions $u(x,y) = xy$ over 2 commodities. The endowment of a type $i \in \{2,4,6,\dots,n-1\}$ agent is (3,1), and the endowment of a type $j \in \{3,5,7,\dots,n\}$ consumer is (1,3). Type 1 consumers have a utility function $u(x,y) = x+y$ and endowment $(\alpha,\beta) \in \mathbb{R}^2_+$.

Suppose $\frac{n-1}{2}$ markets are active. The access fee of market γ_k, where $k \in \{1,\dots,\frac{n-1}{2}\}$, is $(\phi_k,\phi_k) \in [0,1]^2$. The positive fractions of middlemen at the active markets are formed by type 1 consumers. Let M_k denote the fraction of type 1 middlemen at market k, with $\sum_{k=1}^{\frac{n-1}{2}} M_k = 1$. Every market has a zero set-up cost.

Suppose consumers of type i and $i+1$, $i \in \{2,4,\dots,n\}$, are traders at market $i/2$. Then every consumer at market k trades a net equilibrium bundle $(2-\phi_k, 2-\phi_k)$, as is easily seen. The traders at market k therefore have payoff $4(1-\phi_k)^2$. Individual rationality is satisfied if $4(1-\phi_k)^2 \geq 3$, that is, $\phi_k \leq 1-\sqrt{3}/2$.

The aggregate bundle of a middleman at market k is $\left(\frac{2\phi_k}{M_k}, \frac{2\phi_k}{M_k}\right)$. The candidate equilibrium fractions are found by solving

$$\alpha+\beta+\frac{4\phi_1}{M_1} = \alpha+\beta+\frac{4\phi_2}{M_2} = \dots = \alpha+\beta+\frac{4\phi_q}{M_q}, \text{ with } q = \frac{n+1}{2},$$

which is solved by

$$\hat{M}_k = \frac{\phi_k}{\sum_{j=1}^{q}\phi_j}.$$

We have an equilibrium if $\phi_k \leq 1-\sqrt{3}/2$ for every k, and no consumer has an incentive to become a middleman, that is

$$\left(3+2\sum_{j=1}^{q}\phi_j\right)\left(1+2\sum_{j=1}^{q}\phi_j\right) \leq 4(1-\phi_k)^2 \text{ for all } k \in \{1,\dots,q\}.$$

As an illustration of this last constraint, suppose $\phi_1 = \phi_2 = \dots = \phi_q =: \phi$. Then, an equilibrium exists if

$$\phi \leq \frac{\sqrt{3+8q+5q^2}-2(1+q)}{4(q^2-1)}.$$

For $n = 3$ and $q = 2$, the upperbound is $1/16$. The upperbound tends to zero if n, and hence q, tends to infinity.

$\square$

The number of equilibria of the form illustrated in the example is in principle very large. This is a result of the inertia associated with a continuum of traders. That is, traders of a zero fraction are not 'recognized' on markets. The equilibria from Theorem 2.3.6 have the property that all traders of the same type select the same market, so that that type is represented with a zero fraction at any other market. Therefore, no trader has an incentive to deviate to a different market, having to consume his reduced endowment. In the example illustrating the theorem, numerous other distributions of traders over markets would have sustained an equilibrium, if the number of active markets would be smaller.

Notice that the inertia does not occur for the middlemen. Middlemen may well have an incentive to deviate to a different active market individually, as they are treated anonymously with respect to the division of access fees. Whereas a zero fraction of traders is not 'recognized' on the market, a zero fraction of middlemen joining an existing group of middlemen at an active market shares in the collected access fee. Middlemen are discriminated only to the extent that set-up costs may be different for different types.

The inertia for traders would disappear in case consumers form a countable population. In that case, individual agents, having measure zero in case of a continuum, are able to trade, and therefore may have an incentive to switch markets. We illustrate this in the following example.

Example 2.3.8 Consider a distribution where four consumer types disperse over two markets as traders as follows: type 1 and 2 agents are traders at market 1, whereas type 3 and 4 agents are traders at market 2. Endowments and utility functions are as follows:

$$w^1 = w^3 = (1,0),\; w^2 = w^4 = (0,1)$$

$$u^1(x,y) = u^3(x,y) = y,\; u^2(x,y) = u^4(x,y) = x.$$

There are $t \in \mathbb{N}$ traders of each type.

The access fee of market $j \in \{1,2\}$ is a proportion $\phi_j \in (0,1)$ of endowments. In the competitive equilibrium at market $k \in \{1,2\}$, total supply $(1-\phi_k)t$ of good $i \in \{1,2\}$ is divided between the traders of type $j \neq i$. The given distribution can be sustained as an equilibrium if no trader has an incentive to deviate to the other market, that is,

$$\frac{t}{t+1}(1-\phi_2) \leq 1-\phi_1 \text{ and } \frac{t}{t+1}(1-\phi_1) \leq 1-\phi_2.$$

Deviation from the market with the high to the low access fee is not necessarily profitable, since terms-of-trade are decreased. In the limit case $t \to \infty$ an equilibrium can be sustained only if $\phi_1 = \phi_2$. Then, if $\phi_1 < \phi_2$, deviation from market 2 to market 1 is always profitable. This is because terms-of-trade are not decreased; the access fee is the only relevant variable for a deviating trader.

The limit case $t \to \infty$ does not correspond to an economy with a continuum of consumers. There, a distribution with all type 1 and 2 traders at market 1 and all type 3 and 4 traders at market 2 could be sustained as an equilibrium for any access fees.

□

The example illustrates how the inertia associated with a continuum of traders could be solved by considering a countable number of consumers. There is a different kind of inertia, which cannot be solved in this way, namely the inertia associated with the effective activation of non-active markets. In order to activate a market effectively, that is, having consumers extract gains-from-trade, a positive fraction of middlemen and positive fractions of at least two different types of traders are needed. Were the number of consumers countable, we would need at least three consumers: one middlemen and two traders of different types. This kind of inertia may lead to Pareto-inferior lock-in of consumers into markets. It may happen that all consumers coordinate on a market with a high access fee, though it is a Pareto improvement if they coordinate on a market with a lower fee.

The second existence theorem considers the case where the set-up costs of active markets are not necessarily small. Then, there must be some type whose initial endowments are small relative to the other types' endowments.

Theorem 2.3.9 *Let be given numbers $\delta > 0$ and $\epsilon > 0$, a type $k \in N$, and a non-empty subset $\Gamma^* \subset \Gamma$ with $\#\Gamma^* \leq n/2$ such that for all $\gamma \in \Gamma^*$ their market characteristics $\psi(\gamma) = (\sigma, \phi)$, and all endowments w^i, $i \in N \setminus \{k\}$ have the property that*

1. $||\phi^i|| < \delta$ *for all* $i \in N$.
2. $\frac{||w^k||}{||w^i||} < \epsilon$.

If δ and ϵ are sufficiently small, then there generically exists a market equilibrium $\hat{s} \in \Delta$ with the property that all markets $\gamma \in \Gamma^*$ are active.

For the proof, we refer to Section 2.6. We illustrate the structure of the equilibrium constructed in the proof in the following example.

Example 2.3.10 Given is an economy with two types and two commodities. Each type has a Cobb-Douglas utility function $u(x, y) = xy$. The initial endowment of type 1 is $(\alpha_1, 0)$, $\alpha_1 > 0$, whereas type 2 has endowment (α_2, β_2), $\alpha_2, \beta_2 > 0$.

Consider a market $\gamma \in \Gamma$ with access fee in the form of a proportion of endowments $\phi \in (0, 1]$, and a set-up cost $(\sigma^1, \sigma^2) = ((\alpha_1, 0), (\alpha_2, \beta_2))$ equal to entire endowments. The payoffs of middlemen from positive fractions are therefore determined completely by the collected access fees, so that

$$f^i_{\gamma m}(s) = \frac{\phi^2(\alpha_1 s^1_{\gamma c} + \alpha_2 s^2_{\gamma c})\beta_2 s^2_{\gamma c}}{(s^1_{\gamma m} + s^2_{\gamma m})^2}, i \in \{1, 2\}.$$

We normalize the price of the first commodity to one. Then, given $s \in \Delta$, at market γ an equilibrium price of the second commodity

$$p_\gamma(s) = \frac{\alpha_1 s^1_{\gamma c} + \alpha_2 s^2_{\gamma c}}{\beta_2 s^2_{\gamma c}}$$

is generated. The traders' payoffs are

$$f^1_{\gamma c}(s) = \frac{(\alpha_1)^2}{4p_\gamma(s)} \text{ and } f^2_{\gamma c}(s) = \frac{(\alpha_2 + \beta_2 p_\gamma(s))^2}{4p_\gamma(s)}.$$

For the economy in this example, an equilibrium s constructed in the proof of Theorem 2.3.9 satisfies $0 < s^i_{\gamma m} = 1 - s^i_{\gamma c} < 1$ and $s^j_{\gamma c} = 1$, $i \neq j \in \{1,2\}$. We have such an equilibrium if, first, $f^i_{\gamma m}(s) = f^i_{\gamma c}(s)$, and, secondly, $f^j_{\gamma c}(s) \geq f^j_{\gamma m}(s)$.

For $i = 1$, we find a candidate equilibrium $\hat{s}$ with

$$\hat{s}^1_{\gamma m} = \frac{2(\alpha_1 + \alpha_2)\phi}{\alpha_1(1+\phi)} \text{ and } \hat{s}^1_{\gamma c} = \frac{\alpha_1 - (\alpha_1 + 2\alpha_2)\phi}{\alpha_1(1+\phi)},$$

provided $\phi < \frac{\alpha_1}{\alpha_1 + 2\alpha_2}$. For individual rationality we need

$$(1-\phi)^2 \frac{(\alpha_2 + \beta_2 p_\gamma(\hat{s}))^2}{4p_\gamma(\hat{s})} \geq \alpha_2\beta_2 \text{ or } \phi \leq \frac{\alpha_2 + \beta_2 p_\gamma(\hat{s}) - 2\sqrt{\alpha_2\beta_2 p_\gamma(\hat{s})}}{\alpha_2 + \beta_2 p_\gamma(\hat{s})}.$$

The second equilibrium condition is equivalent to $f^1_{\gamma c}(\hat{s}) \leq f^2_{\gamma c}(s)$, which boils down to $\alpha_1 \leq \alpha_2/\phi$.

If the initial endowment of type 1 is small relative to the endowment of type 2, type 2 consumers do not have an incentive to become a middleman.

□

We conclude this section with the observation that for the limit case of zero access fees for all active markets, the zero equilibrium coalitions of middlemen could be considered as a set of auctioneers, each of which serves a local, costless, Walrasian market.

2.4 Strong equilibria

In the previous section, a large number of market equilibria was shown to be sustained. This multitude is caused by two kinds of inertia. First, inertia associated with a continuum of traders; traders being members of zero fractions are not 'recognized' on markets. Second, inertia caused by the inability of coalitions of zero measure to activate non-active markets effectively. In this section, both types of inertia are 'resolved' by incorporating the possibility of coalitions deviating. We consider coalitional stability of market equilibria using the concept of strong equilibrium as introduced by Aumann (1960)[2]. A market equilibrium is strong if no coalitions of consumers has an incentive to deviate.

The choices of roles and markets by consumers of a subset $B \subset A$ is described by a *profile* $\pi_B : B \to \Sigma$, describing the distribution of middlemen and consumers over markets. For a given profile π_B, every consumer $a \in B$ has a payoff $g(a, \pi_B)$.

Definition 2.4.1 *A profile $\pi_A : A \to \Sigma$ is* **blocked** *by a coalition $B \subset A$ and a profile $\pi_B : B \to \Sigma$ if $g(a, \pi_B) > g(a, \pi_A)$ for every $a \in B$.*

We observe that each state vector $s \in \Delta$ is linked to a profile π_A^s such that for every $i \in N$ and $(\gamma, r) \in \Sigma$

$$s_{\gamma r}^i = \mu(\{a \in A^i \mid \pi_A^s(a) = (\gamma, r)\}).$$

Definition 2.4.2 *A state $s \in \Delta$ is a* **strong equilibrium** *if its associated profile π_A^s is not blocked by any coalition $B \subset A$ and profile π_B.*

It is immediately clear, that every strong equilibrium is Nash, since otherwise its profile would be blocked by some coalition consisting of one consumer. Though strong equilibrium is a refinement of Nash equilibrium (van Damme (1991)), it has also a cooperative interpretation; it considers deviations by coalitions whose members coordinate actions amongst each other. In a strong equilibrium, the assignment of consumers to roles and markets is such that no coalition is able to find an assignment that yields all its members a strict payoff improvement. In particular, the 'grand coalition' A cannot. Strong equilibria yield therefore an efficient division of payoffs, given the institutional structure of markets supporting the economy. Were roles and markets assigned by a social planner, strong equilibrium profiles are candidate assignments.

As a first, straightforward, result we find that a no-trade equilibrium, in which all markets are non-active, is not strong if there is a subset of markets with sufficiently small access fees.

[2]This concept of strong equilibrium is different from the strong equilibrium concept introduced by Harsanyi (1973).

Proposition 2.4.3 *Let be given a number $\delta > 0$, a non-empty subset of markets $\Gamma^* \subset \Gamma$, and a non-empty subset of types $M \subset N$ with $\#M \geq 2$, such that every market $\gamma \in \Gamma^*$ with characteristic $(\sigma(\gamma), \phi(\gamma)) \in \Gamma^*$ satisfies $||\phi^i(\gamma)|| < \delta$ for all $i \in M$.*

Suppose $s \in \Delta$ is a strong equilibrium. If δ is sufficiently small, then s is a market equilibrium.

PROOF

Suppose s is not a market equilibrium, then every market $\gamma \in \Gamma$ is either non-active, or is active with only one type of trader represented by a positive fraction. Hence, all consumers have a payoff less than or equal to the utility of initial endowments. For types $i, j \in M$ with $j \neq i$, let $\eta \subset A^i$ be a coalition of middlemen, and $\theta_i \subset A^i$ and $\theta_j \subset A^j$ be coalitions of traders, where $\mu(\eta), \mu(\theta_i), \mu(\theta_j) > 0$. Suppose the coalition $\eta \cup \theta_i \cup \theta_j$ deviates to a market $\gamma \in \Gamma^*$, then the traders' payoffs become strictly larger than the utility of their initial endowments if the access fees ϕ^i and ϕ^j are sufficiently small. The middlemen strictly improve if $\mu(\eta)$ is sufficiently small, as middlemen's payoffs tend to infinity if $\mu(\eta)$ tends to zero.

□

The bound on access fees is determined by gains-from-trade obtainable, as illustrated in the following example.

Example 2.4.4 Consider an economy with two types and two commodities, quantities of which are denoted by x and y, respectively. Type 1 consumers have a utility function $u^1(x, y) = x$ and an endowment $(\alpha_1, \beta_1) \in \mathbb{R}^2_+$, whereas type 2 consumers have a utility function $u^2(x, y) = y$ and an endowment $(\alpha_2, \beta_2) \in \mathbb{R}^2_+$. Let $\gamma \in \Gamma$ be a market with access fee $\phi^i = \phi \cdot (\alpha_i, \beta_i)$ for type $i \in \{1, 2\}$, with $\phi \in (0, 1)$ a fraction of endowments.

Depart from the situation in which all markets are non-active. Suppose a coalition of positive measure consisting of middlemen and traders from both types deviates to market γ. If the measure of the group of middlemen is sufficiently small with respect to the measure of the group of traders, they increase their payoffs strictly above the utility of their initial endowments. The traders have a strict increase if

$$\begin{cases} (1-\phi)\frac{\alpha_1 C_1+\alpha_2 C_2}{C_1} > \alpha_1 \\ \\ (1-\phi)\frac{\beta_1 C_1+\beta_2 C_2}{C_2} > \beta_2, \end{cases} \tag{2.2}$$

where $C_1, C_2 > 0$ are the measures of the deviating trader coalitions from type 1 and 2, respectively. In the Walrasian equilibrium, the aggregate net endowments of commodity $i \in \{1,2\}$ are allocated to type i traders entirely in the Walrasian equilibrium.

The system (2.2) boils down to

$$\frac{\phi\alpha_1}{(1-\phi)\alpha_2} < \frac{C_2}{C_1} < \frac{(1-\phi)\beta_1}{\phi\beta_2},$$

which yields feasible C_1 and C_2 only if

$$\frac{\phi\alpha_1}{(1-\phi)\alpha_2} < \frac{(1-\phi)\beta_1}{\phi\beta_2} \text{ or } \phi < \frac{\sqrt{\alpha_2\beta_1}}{\sqrt{\alpha_1\beta_2}+\sqrt{\alpha_2\beta_1}}.$$

The upperbound on ϕ is decreasing in the product $\alpha_2\beta_1$, which indicates how large gains-from-trade are.

□

Notice that for the coalition activating a market, only relative sizes of groups of traders and middlemen matter. An arbitrary small coalition may activate a market, which makes this likely to happen, as communication requirements within a coalition decrease with its size.

Strongness properties of market equilibria, in which gains-from-trade are realized, are far from straightforward. The reason is that the reaction to changes in access fees of competitive equilibria is depending heavily on consumers' endowments and utility functions. It is not true in general, for instance, that deviation to a market with a smaller access fee yields all traders a higher payoff. Although at least one type of trader obtains a larger equilibrium bundle if net endowments become larger, it could well happen that gains-from-trade decrease for the other traders. Not being able to derive more general results, we illustrate the strongness properties of economies with Cobb-Douglas utility functions. We consider the case of n types and 2 commodities.

Example 2.4.5 The economy has two commodities. Suppose $u^i(x,y) = xy$ and $w^i = (\alpha_i, \beta_i)$ for every $i \in N$.

The set of markets is $\{\gamma_0, \gamma_1, \ldots, \gamma_q\}$, where market γ_k, $1 \leq k \leq q$, has a proportional access fee $(\phi_k, \phi_k)w^i$ for type $i \in N$, and $\phi_0 = 0$. The access fees are such that $0 < \phi_1 < \phi_2 < \ldots < \phi_q$.

If the price of the first commodity is normalized to one at all markets, type i's demand for the first commodity at an active market γ_k, given a price $p > 0$ of the second commodity, is $d^i_{1k}(p) = (1-\phi_k)\frac{\alpha_i+\beta_i p}{2}$, and $d^i_{2k}(p) = (1-\phi_k)\frac{\alpha_i+\beta_i p}{2p}$ for the second commodity.

Solving the market clearing condition $\sum_{i\in N} d^i_{1k}(p)s^i_{\gamma_k c} = \sum_{i\in N}(1-\phi_k)\alpha_i s^i_{\gamma_k c}$ yields the equilibrium price of the second commodity at market γ_k

$$p_k(s) = \frac{\sum_{i\in N}\alpha_i s^i_{\gamma_k c}}{\sum_{j\in N}\beta_j s^j_{\gamma_k c}}.$$

Notice that the proportional access fee ϕ_k does not influence $p_k(s)$.

The payoff of a type i trader at market γ_k is

$$f^i_{\gamma_k c}(s) = (1-\phi_k)^2\frac{(\alpha_i + \beta_i p_k(s))^2}{4p_k(s)}.$$

The payoff of a type i middleman at market γ_k is

$$f^i_{\gamma_k m}(s) = (\phi_k)^2\frac{\left(\sum_{j\in N}\alpha_j s^j_{\gamma_k c}\right)\left(\sum_{j\in N}\beta_j s^j_{\gamma_k c}\right)}{\left(\sum_{j\in N} s^j_{\gamma_k m}\right)^2}.$$

We show that a market equilibrium is strong only if all consumers go to market γ_1, having the smallest positive access fee. Suppose therefore that a strong market equilibrium s exists with market γ_k, $k > 1$, active. Suppose $s^i_{\gamma_k m} > 0$ for all $i \in M_k \subset N$, and $s^j_{\gamma_k c} > 0$ for all $j \in C_k \subset N$. Suppose first market γ_1 is not active. Consider a deviating coalition $\delta = (\cup_{i\in M_k}\eta_i)\cup(\cup_{j\in C_k}\theta_j)$ of middlemen (η_i) and traders (θ_j) going to market γ_1, satisfying $\mu(\theta_j) = s^j_{\gamma_k c}$ for all $j \in C_k$, and $0 < \sum_{i\in M_k}\mu(\eta_i) < \frac{\phi_1\sum_{j\in M_k} s^j_{\gamma_k m}}{\phi_k}$.

In case the coalition δ deviates, the equilibrium price $\tilde{p}_1(\delta)$ of the second commodity at γ_1 is $p_k(s)$. Therefore, all traders from $\cup_{j\in C_k}\theta_j$ have a strict payoff improvement as $\phi_1 < \phi_k$. As the measure of the deviating middleman coalition is chosen sufficiently small, all middlemen also have a strict payoff improvement.

Suppose on the other hand that γ_1 is active. Suppose $p_k(s) > p_1(s)$. Consider a deviation from market γ_k to γ_1 of a coalition of type $i^* \in C_k$ traders of size $\delta > 0$ to market γ_1, where i^* is such that

$$\frac{\alpha_{i^*}}{\beta_{i^*}} = \max_{i\in C_k}\left\{\frac{\alpha_{i^*}}{\beta_{i^*}}\right\}.$$

Let the price of the second commodity after deviation be $\tilde{p}_1(\delta)$, with δ sufficiently small to ensure $p_k(s) > \tilde{p}_1(\delta)$.

The traders from the deviating coalition strictly improve their payoffs if

$$\frac{(\alpha_{i^*} + \beta_{i^*}\tilde{p}_1(\delta))^2}{4\tilde{p}_1(\delta)} > \frac{(\alpha_{i^*} + \beta_{i^*}p_k(s))^2}{4p_k(s)}, \text{ or}$$

$$(\alpha_{i^*})^2(p_k(s) - \tilde{p}_1(\delta)) > (\beta_{i^*})^2\tilde{p}_1(\delta)p_k(s)(p_k(s) - \tilde{p}_1(\delta)).$$

This last condition becomes $\left(\frac{\alpha_{i^*}}{\beta_{i^*}}\right)^2 > \tilde{p}_1(\delta)p_k(s)$, which is satisfied if $p_k(s) < \frac{\alpha_{i^*}}{\beta_{i^*}}$. By the choice of i^*, this is the case. Hence, again deviation is profitable for some coalition.

In case $p_k(s) < p_1(s)$, deviation is profitable for a sufficiently small coalition of type j^* traders, with

$$\frac{\alpha_{j^*}}{\beta_{j^*}} = \min_{j\in C_k}\left\{\frac{\alpha_{j^*}}{\beta_{j^*}}\right\}.$$

The final step is to show that a market equilibrium $\hat{s}$ with all consumers going to market γ_1 is strong. Suppose a coalition containing type $i \in C \subset N$ traders deviates to market $\gamma_k \neq \gamma_1$. Then, the type i^* with $\frac{\alpha_{i^*}}{\beta_{i^*}} = \max_{i\in C}\{\frac{\alpha_i}{\beta_i}\}$, or the type j^* with $\frac{\alpha_{j^*}}{\beta_{j^*}} = \min_{j\in C}\{\frac{\alpha_j}{\beta_j}\}$ has a strictly lower payoff after deviation, which follows from the argument before. This implies that $\hat{s}$ is indeed strong.

In this example, the two kinds of inertia are both solved in a 'desirable' way. First, we have that any deviation of trader fractions to active markets with lower access fees is profitable. Second, if a market with a smaller access fee is non-active, it can be profitably activated.

2.5 Sequential-move market formation: an example

This section gives an example of a sequential move market formation game. It illustrates the sensitivity of the order of play on the configuration of middlemen, traders and fees.

Consider an economy with four consumers and two commodities. Consumer 1 and 2 are of type a; their endowment is $(3, 1)$. Consumer 3 and 4 are of type b; their endowment is $(1, 3)$. Both types have a Cobb-Douglas utility function $u(x, y) = xy$.

We consider the sequential game in which consumers at each stage of the game have the following strategies available.

1. Become a middleman charging a fee proportional to endowments, keeping one's entire endowment, and activating a competitive market.
2. Become a trader with an existing middleman.
3. Consume one's endowment.

The fee is identical for all types of traders, for convenience.

Essentially, there are three different orders of play to be considered, namely (a, a, b, b), (a, b, a, b), and (a, b, b, a). We determine the subgame-perfect Nash equilibrium configuration of middlemen, traders, and access fees. Thereby, we assume that in case the position of trader earns the same payoff as the position of middleman, a consumer will become trader.

(a,a,b,b)

Consider the first order of play. It is clear that only the first mover will become middleman. For suppose the second mover becomes middleman also, then the two b types will consume their endowments, as no gains-from-trade are obtainable. The middleman has two options: either, attract all three other consumers as traders, or only attract the second and third mover. That attracting only the third and fourth mover is never profitable, is obvious.

Suppose the middleman attracts all other consumers. Then, the competitive equilibrium utility of the type a trader is approximately $4.86(1-\phi)^2$, whereas the equilibrium utility of the type b traders is approximately $3.46(1-\phi)^2$. In order to attract all three consumers, the maximal fee ϕ^* is such that $3.46(1-\phi^*)^2 = 3$ (individual rationality), that is, $\phi^* = 0.0688$. This yields the middleman a utility $(3+5\phi^*)(1+7\phi^*) = 4.96$.

Suppose on the other that the middleman attracts only the second and third mover. Then, the competitive equilibrium utilities of both traders are $4(1-\phi)^2$, so that the maximal fee is $\hat{\phi} = 0.134$, yielding the middleman a utility $(3+4\hat{\phi})(1+4\hat{\phi}) = 5.43$. It is thus optimal not to attract the last consumer. The middleman extracts all gains-from-trade.

(a,b,a,b)

In the second order of play, the first mover faces an extra constraint as middleman, namely to prevent the second mover to become middleman also. Suppose the first mover sets a fee ϕ. If the second mover becomes middleman, he will attract the remaining consumers as traders at fee $\phi - \epsilon$, where $\epsilon > 0$ is arbitrarily small. The second mover has than a utility arbitrarily close to $(1+4\phi)(3+4\phi)$. The first-mover has again two options for choosing ϕ. Either, he attracts all remaining players as traders, or he attracts only the second and third mover.

In the first case, the optimal fee ϕ^* follows from $(1+4\phi^*)(3+4\phi^*) = 3.46(1-\phi^*)^2$, or $\phi^* = 0.0199$, yielding the first mover a payoff 3.531. In the second case, the optimal fee $\hat{\phi}$ follows from $(1+4\hat{\phi})(3+4\hat{\phi}) = 4(1-\hat{\phi})^2$, or $\hat{\phi} = 0.0408$, yielding a payoff 3.679.

Hence, the first mover sets a fee 0.0408, attracting the second and third mover as traders. By the threat of the second mover entering as middleman, only part of the gains-from-trade can be extracted by the middleman.

(a,b,b,a)

The third order of play is essentially the same as the second. Now, the third mover is excluded as trader.

In all three cases, the equilibrium configuration has one middleman with two traders. Comparing the payoff distributions, we find that in the first case, the middleman has a payoff 5.43, whereas the two traders have a payoff 3. This distribution of payoffs resembles the uncontested position of the middleman. In the second and third configuration, the middleman and the two traders have all a payoff 3.68. This 'fair' distribution resembles that the position of the middleman is contested by the second mover.

The example shows the sensitivity of outcomes with respect to the order of play. Here, only one middleman can be active profitably. With more consumers, we could have several middlemen active. This would increase the number of potential equilibrium configurations considerably. The sensitivity of configurations to the order of entry is analyzed by Arthur (1990) with respect to firms entering an industry. Increasing-returns from the presence of other firms yields a large path-depency, that is, initial configurations of firms determine largely the long-run outcome. Here, the externality associated with markets causes path-dependency. On the influence of history on locking-in, see also Arthur (1989).

Besides path-depency, the example shows that it may be profitable for a middleman to exclude certain consumers as traders. By doing this, he increases gains-from-trade for the other traders, so that the fee can be higher. Notice that we required the fee to be uniform in this example; no price discrimination is allowed. Also, the threat of other consumers entering as middlemen lowers the fee. In the first order of play, the middleman has effectively a monopoly position, since the other consumers do not have a credible threat to enter as a middleman. In the second and third order, however, the threat of entry is credible, so that the equilibrium fee is lower.

2.6 Proofs

Proof of Theorem 2.3.6

Let $t \in \mathbb{N}$. Let the subspace $\tilde{\Delta} \subset \Delta$ be such that every $s \in \tilde{\Delta}$ satisfies the following conditions.

1. $0 < s^1_{\gamma_j m} < 1$ for every $\gamma_j =\in \Gamma^* = \{\gamma_1, \ldots, \gamma_t\} \subset \Gamma$.
2. For every $1 \leq j \leq t$: $s^i_{\gamma_j c} = 1$ for some $i \neq 1 \in I_j \subset N$, with $\#I_j \geq 2$.

We derive an equilibrium in the space $\tilde{\Delta}$.

For expositional ease, we show the theorem for the case where every $\gamma \in \Gamma^*$ has a characteristic $\psi(\gamma) = (\sigma, \phi)$ such that

$$\sigma^i = 0_l \in X,\ \phi^i_1 > 0, \text{ and } \phi^i_k = 0 \text{ for every } 2 \leq k \leq l, i \in N,$$

thereby assuming that the economy (E_0, Γ) is not an element of some residual subset of $\mathcal{E}$. That means that the result can be generalized for positive and small set-up costs. This assumption is not too restrictive as we only eliminate measure zero cases.

By Lemmas 2.2.7 and 2.2.9 the result is generalized for positive and small set-up costs, and positive access fees.

Derivation of a candidate equilibrium

For notational convenience, we introduce for market γ_j, $1 \leq j \leq t$, with access fee of the first commodity phi^i_{j1}, $i \in N$,

$$M_j := s^1_{\gamma_j m} \text{ and } \alpha_j := \sum_{i \in I_j} \phi^i_{j1}.$$

In order to derive a candidate equilibrium in $\tilde{\Delta}$, we solve $f^1_{\gamma_1 m}(s) = f^1_{\gamma_2 m}(s) = \ldots = f^1_{\gamma_t m}(s)$, which is equivalent to

$$u^1\left(\frac{\alpha_1}{M_1}+w_1^1,w_2^1,\ldots,w_l^1\right)=\ldots=u^1\left(\frac{\alpha_t}{M_t}+w_1^1,w_2^1,\ldots,w_l^1\right).$$

By strict monotonicity of u^1, this condition is equivalent to

$$\frac{\alpha_1}{M_1}=\ldots=\frac{\alpha_t}{M_t},$$

which yields

$$M_j=\frac{\alpha_j}{\alpha_1}M_1, j\in\{2,\ldots,t\}. \tag{2.3}$$

Solving $\sum_{j=1}^t M_j=1$, we obtain candidate equilibrium fractions

$$\hat{M}_j=\frac{\alpha_j}{\sum_{k=1}^t\alpha_k}, 1\leq j\leq t.$$

No-deviation conditions

The second equilibrium condition is individual rationality. For the type 1 middlemen, this is obviously satisfied. For the traders, we derive by Lemma 2.2.13 and Lemma 2.2.14 the existence of a bound $\delta_1>0$ such that $f^i_{\gamma_j c}(\hat{s})\geq u^i(w^i)$ whenever $\phi^1_{j1}<\delta_1$ for every $i\in I_j$, $1\leq j\leq t$.

The third equilibrium condition is that traders do not have an incentive to become middleman, that is, $f^i_{\gamma_j c}(\hat{s})\geq f^i_{\gamma_k m}(\hat{s})$ for all $i\in I_j, 1\leq j,k\leq t$. For the limit case $\phi^i_{j1}=0$ for all $i\in I_j, 1\leq j\leq t$, this condition is obviously satisfied. By continuity, we therefore are able to find a bound $\delta_2>0$ such that the condition holds whenever $0<\phi^i_{j1}<\delta_2$ for every $i\in I_j, 1\leq j\leq t$.

We conclude that there exists a bound $\delta=\min\{\delta_1,\delta_2\}$ such that if $\phi^i_{j1}<\delta$ for all $i\in I_j, 1\leq j\leq t$, an equilibrium market structure $\hat{s}$ exists with markets γ_1 till γ_t active.

□

Proof of Theorem 2.3.9

Let $t\in\mathbb{N}$. Let $\widetilde{\Delta}\subset\Delta$ be such that every $s\in\widetilde{\Delta}$ satisfies the following conditions.

1. $0 < s^1_{\gamma_j m} < 1$ for every $\gamma_j \in \Gamma^* = \{\gamma_1, \dots, \gamma_t\} \subset \Gamma$.

2. $s^1_{\gamma_1 c} = 1 - \sum_{j=1}^t s^1_{\gamma_1 m}$.

3. For every $1 \leq j \leq t$: $s^i_{\gamma_j c} = 1$ for some $i \neq 1 \in I_j \subset N$, with $\#I_j \geq 2$.

We derive an equilibrium in the collection $\widetilde{\Delta}$. For expositional ease, we show the theorem for the case where every market $\gamma_j \in \Gamma^*$ has a characteristic such that

$$\sigma^i_j = (\sigma, \dots, \sigma) \in X,\ \phi^i_{j1} > 0,\ \text{and}\ \phi^i_{jk} = 0 \text{ for every } 2 \leq k \leq l, i \in N.$$

Also, the economy (E_0, Γ) is not part of a residual subset of $\mathcal{E}$.

Derivation of a candidate equilibrium

We introduce

$$M_j := s^1_{\gamma_j m},\ C := s^1_{\gamma_1 c},\ \alpha_j := \sum_{i \in I_j} \phi^i_{j1},\ 1 \leq j \leq t,\ \text{and}\ \phi := \phi^1_{11}.$$

In order to derive a candidate equilibrium in $\widetilde{\Delta}$, we solve

$$f^1_{\gamma_1 m}(s) = f^1_{\gamma_2 m}(s) = \dots = f^1_{\gamma_t m}(s) = f^1_{\gamma_1 c}(s). \tag{2.4}$$

which is equivalent to

$$u^1\left(\frac{\alpha_1 + \phi C}{M_1} + w^1_1 - \sigma, \bar{w}\right) = \dots = u^1\left(\frac{\alpha_2}{M_2} + w^1_1 - \sigma, \bar{w}\right) =$$

$$u^1\left(\frac{\alpha_t}{M_t} + w^1_1 - \sigma, \bar{w}\right),$$

with $\bar{w} = (w^1_2 - \sigma, \dots, w^1_l - \sigma)$. By strict monotonicity of u^1, this condition is equivalent to

$$\frac{\alpha_1 + \phi C}{M_1} = \frac{\alpha_2}{M_2} = \dots = \frac{\alpha_t}{M_t},$$

which yields

$$M_j = \frac{\alpha_j}{\alpha_1 + \phi C} M_1, j \in \{2, \ldots, t\}, \text{ or} \tag{2.5}$$

$$\sum_{j=2}^{t} M_j = \frac{M_1}{\alpha_1 + \phi C} \sum_{j=2}^{t} \alpha_j.$$

Substituting

$$C = 1 - M_1 - \sum_{j=2}^{t} M_j \tag{2.6}$$

we solve

$$\sum_{j=2}^{t} M_j = \frac{\alpha_1 + \phi(1 - M_1) - \sqrt{(\alpha_1 + \phi(1 - M_1))^2 - 4\phi \sum_{j=2}^{t} \alpha_j M_1}}{2\phi}.$$

The necessary condition $\sum_{j=2}^{t} M_j < 1 - M_1$ is satisfied if

$$0 < M_1 < \frac{\alpha_1}{\sum_{j=1}^{t} \alpha_j} =: \beta. \tag{2.7}$$

We show that on $\widetilde{\Delta}$ a solution $\hat{M}_1 \in (0, \beta)$ can be found to the equation

$$f^1_{\gamma_1 c}(s) = u^1 \left(\frac{\alpha_1 + \phi C}{M_1} + w^1_1 - \sigma, \bar{w} \right) \tag{2.8}$$

Consider the following limits on $\widetilde{\Delta}$.

$$L_1(\gamma_1) := \lim_{M_1 \downarrow 0} f^1_{\gamma_1 m}(s) = \infty > u^1(w^1), \text{ and}$$

$$L_2(\gamma_1) := \lim_{M_1 \uparrow \beta} f^1_{\gamma_1 m}(s) = u^1 \left(\frac{\alpha_1}{\beta} + w^1_1 - \sigma, \bar{w} \right).$$

Moreover, for any $s \in \widetilde{\Delta}$,

$$f^1_{\gamma_1 c}(s) \leq u^1 \left(w^1 + \frac{\sum_{i \in I_1} w^i}{C} \right),$$

since a type 1 trader could not obtain more than a proportional part of the aggregate endowment in market γ_1 as a whole.

Therefore,

$$L_3(\gamma_1) := \lim_{M_1 \downarrow 0} f^1_{\gamma_1 c}(s) \leq u^1 \left(w^1 + \sum_{j \in I_1} w^j \right) < \infty.$$

Finally, define

$$L_4(\gamma_1) := \lim_{M_1 \uparrow \beta} f^1_{\gamma_1 c}(s) > u^1(w^1_1 - \phi, w^1_2, \ldots, w^1_l).$$

We have $L_1(\gamma_1) > L_3(\gamma_1)$, and $L_2(\gamma_1) < L_4(\gamma_1)$ if $\phi < \frac{\beta}{\beta + \alpha_1} =: \delta_1$.

By continuity of the payoff functions, we can apply the Intermediate Value Theorem on the interval $[\eta, \beta - \eta]$ for $\eta > 0$ and sufficiently small, to conclude that for all $\phi < \delta_1$, there exists a state $\hat{s} \in \widetilde{\Delta}$ satisying (2.4).

No-deviation conditions

The second equilibrium condition is individual rationality. This is satisfied byLemma 2.2.13 and Lemma 2.2.14 whenever $\phi^i_{j1} < \delta$ for all $1 \leq j \leq t, i \in I_j$, for some bound $\delta > 0$ sufficiently small

The third equilibrium condition is that $f^i_{\gamma_j c}(\hat{s}) \geq f^i_{\gamma_z m}(\hat{s})$ for all $i \in I_j, 1 \leq j, z \leq t$.

Therefore, consider the limit case $w^1 = 0_l \in X$. Then, the competitive equilibrium bundle of the type 1 traders is obviously zero. Hence, $f^1_{\gamma_1 c}(\hat{s}) = u^1(0_l)$. Thus, also $f^1_{\gamma_j m}(\hat{s}) = u^1(0_l)$ for all $1 \leq j \leq t$. Hence, the middlemen at market γ_j obtain the zero bundle, implying $f^i_{\gamma_j m}(\hat{s}) = u^i(w^j_1 - \sigma, \ldots, w^j_l - \sigma) < f^i_{\gamma_z c}(\hat{s})$, $1 \leq k \leq t$.

Continuity of the payoff functions implies that there exist an $\epsilon > 0$ sufficiently small, so that $\frac{w_k^1}{w_k^i} < \epsilon$ for all $1 \leq k \leq l, 2 \leq i \leq n$.

We conclude that there exist bounds $\delta > 0$ and $\epsilon > 0$ such that if $\phi_1^i < \delta$ for all $1 \leq j \leq t$, and $\frac{w_k^1}{w_k^i} < \epsilon$ for all $1 \leq k \leq l, 2 \leq i \leq n$, a market equilibrium $\hat{s}$ exists with markets γ_1 till γ_t active.

$\square$

Chapter 3

Market Formation in a Two Region Economy

3.1 Introduction

In this chapter, we analyze the market formation game as introduced in Chapter 2 in the context of an economy with two spatially separated regions. There are two types of consumers and two tradeable commodities for which both types have Cobb-Douglas utility functions with equal weights. A type $i \in \{a, b\}$ consumer has an endowment with a positive quantity of one of the commodities only. All type a consumers are living in region A, whereas all type b consumers are living in region B. In both regions, a market is located. The regions are spatially separated in the sense that there is a transportation cost between the regions. If a type a consumer goes to the market in region B, or a type b consumer to the market in region A, he incurs this transportation cost.

Every consumer may either become *middleman* or *trader*. Positively measured coalitions of middlemen are able to activate markets. Traders take their endowments to the market, exchange endowments for Walrasian equilibrium bundles, and return to their own region to consume these. A type $i \in \{a, b\}$ consumer may become a middleman only in the region he inhabits. Only traders travel between regions. The restriction that middlemen do not leave their region, may have several interpretations. For instance, markets possess a high degree of region specificity, in the sense that learning the market customs and techniques requires too much effort for middlemen from the other region. Another interpretation is that it is legally not possible to become a middleman in the other region. This interpretation would make particularly sense if we think of the regions as countries. In that case, 'human capital' in terms of organizational skills is immobile. The interpretation of the regions as countries links our model somewhat to the Ricardian model of international trade. The endowments with a positive amount of only one commodity could then be interpreted as specialization in one commodity by a country.

The set-up of a two-region economy with transportation costs shares similarities with the models on city-formation as considered in the urban economics literature[1]. In the analogy of those models, a monocentric market equilibrium is similar to a *Central Business District* (CBD). A CBD may be defined as an agglomeration of traders (Papageorgiou and Smith (1983)) or as a concentration of trade (Baesemann (1977)). According to the latter approach, a CBD is a compact subset of a plane or a line segment where all transactions take place. Whereas the Central Business District is a commonly used concept in the urban economics literature, also cities with more than one Business District are considered, for instance see Krugman (1991,1993), and Fujita and Ogawa (1982).

Dispersion of traders over markets is caused by the transportation cost. For relatively small transportation costs, an equilibrium with concentrated trade results, in which all agents go to the same region. The model in this chapter differs from the mentioned urban economics models to the extent that in those models, consumers migrate towards profitable locations in order to trade. In our model, consumers travel to a market as traders, and return to their region afterwards to consume their competitive bundles.

We find three types of Nash equilibria of the game in which all consumers choose a role and a market simultaneously. First, there are *no-trade* equilibria. If there is no trade at a market, either the market is not active, or only one type of trader is represented with a positive fraction. The existence of no-trade equilibria is independent of the sizes of access fees and transportation costs. This is because of the lock-in effect associated with markets; no individual consumer, having measure zero, is able to activate and use markets profitably. A coalition of positive measure consisting of middlemen and traders of both types is needed to do this.

[1] Fujita (1990) provides a survey of urban economics theory.

Second, we have equilibria with *concentrated trade*, in which case one market is active and has two positive fractions of traders, whereas the other market is inactive. Among them, we can distinguish further between equilibria with all consumers being middleman or trader at the active market, and equilibria in which a certain fraction of consumers 'stays home', that is, goes to the inactive market. The first situation is encountered if the transportation cost between the region with the inactive and with the one active market is relatively small. In that case, all consumers from the region with the inactive market are able to bear the transportation cost incurred by travelling to the active market and still have positive net gains-from-trade. If transportation costs are relatively large, we have the situation in which part of these consumers withdraws from the market. Doing this, they improve the position of the other consumers of their type who become trader at the active market. Consumers withdraw until transportation costs and gains-from-trade are balanced. Then, the net gains-from-trade of all consumers from the region with the inactive market are zero.

Third, we have equilibria with *dispersed trade*. In such an equilibrium, both markets have trade and all fractions are strictly positive. This equilibrium exists if transportation costs are relatively large. Hence, dispersion of trade between different markets is caused by increasing transportation costs.

The equilibrium set is considerably large. This is caused by the inability of individual agents, having measure zero, to activate markets. Therefore, all kinds of counter-intuitive equilibria are sustained. For instance, a continuum of no-trade equilibria is found. Although it is favorable to activate markets in order to extract gains-from-trade from the economy, all consumers are locked-in. The lock-in problem is addressed by applying the concept of *strong* equilibrium. An equilibrium is strong if not only individuals do not have an incentive to deviate, but also coalitions of positive measure do not have this incentive.

The strongness criterion eliminates first of all the equilibria in which all, or part of the consumers have the utility of their initial endowments. That is, the no-trade equilibria and the equilibria with concentrated trade in which part of the consumers goes to the inactive market. Those are not strong because certain coalitions of middlemen and traders are able to activate and use an inactive market profitably. Second, an equilibrium with concentrated trade is strong under certain conditions on access fees and transportation cost; the transportation cost incurred from travelling from one region to the other should be sufficiently small, for instance. Also, the set-up cost of the market in the latter region should be sufficiently large. Finally, the equilibrium with dispersed trade is always strong.

The remainder of this chapter is organized as follows. In Section 2, the model of two region economy is formulated. In Section 3, all market equilibria of the market formation game are derived. Section 4 selects the strong equilibria from the set of market equilibria.

3.2 The Model

Consider an economy with two spatially separated regions, A and B. Region A is inhabited by a continuum of type a agents, whereas region B is inhabited by a continuum of type b agents. Both types have equal Lebesgue measures, and are therefore represented both by the unit interval [0,1]. There are two commodities. Type a agents have an initial commodity endowment $(w_a, 0) \in \mathbb{R}^2_+$, and also an amount $L_a > 0$ of some non-tradeable commodity. This commodity could be leisure, for instance, and can be used to travel between regions. Type b consumers have an initial commodity endowment $(0, w_b) \in \mathbb{R}^2_+$, and an amount $L_b > 0$ of non-tradeable commodity. All consumers have a separable utility function over the tradeable and non-tradeable commodities

$$u(x, y, l) = xy + v(l),$$

where $x \geq 0$ and $y \geq 0$ denote quantities of commodity 1 and 2, respectively, and $v(.)$ is strictly increasing and continuous in the quantity $l \geq 0$ of the non-tradeable good.

There are two markets, one in each region. *Middlemen* may activate markets, that is, they provide facilities for the other agents, the *traders*, to trade their endowment bundles. Any consumer from region $j \in \{A, B\}$ may adopt the role of middleman in region j. Type a consumers are not able to become middleman in region B, and type b consumers not in region A. In order to become middleman in region A, a *set-up cost* $(\sigma_A, 0)$ with $0 \leq \sigma_A \leq w_a$ in terms of commodities is incurred, which is subtracted from w_a. Analogously, middlemen in region B incur a set-up cost $(0, \sigma_B)$ with $0 \leq \sigma_B \leq w_b$. The activated markets are used by the other consumers, the *traders*. Type a and b consumers becoming trader at the market in region $i \in \{A, B\}$ pay bundles $\phi_i(w_a, 0)$ and $\phi_i(0, w_b)$, respectively, to the middlemen at that market. The number $\phi_i \in (0,1)$ is the relative *access fee* of market i, a proportion of endowments.

The markets in both regions are competitive. After all consumers have adopted a market, at market i the net commodity endowments $(1-\phi_i)(w_a,0)$ and $(1-\phi_i)(0,w_b)$ of the consumers of type a and b, respectively, are exchanged for competitive equilibrium bundles. The markets are local, in the sense that the equilibrium bundles at market i are depending entirely on the composition of trader types at market i. We normalize the local price of the first commodity to one at both markets, and denote by p_j the price of the second commodity at market $j \in \{A,B\}$.

For price p_j, type a consumers at market j have demanded bundles

$$x_{aj}(p_j) = \left(\frac{(1-\phi_j)w_a}{2}, \frac{(1-\phi_j)w_a}{2p_j}\right),$$

while type b consumers at market j have demanded bundles

$$x_{bj}(p_j) = \left(\frac{(1-\phi_j)w_b p_j}{2}, \frac{(1-\phi_j)w_b}{2}\right).$$

Let $0 \leq M_i \leq 1$ denote the fraction of consumers in region i becoming middleman and let $0 \leq C_{ij} \leq 1$ denote the fraction of consumers of type i becoming traders at the market in region j. We have

$$M_A + C_{aA} + C_{aB} = M_B + C_{bA} + C_{bB} = 1.$$

Distributions of agents over roles and regions are denoted by a *state*

$$s := ((M_A, C_{aA}, C_{aB}), (M_B, C_{bA}, C_{bB}))$$

out of the state space Δ.

At the competitive market at region j, an equilibrium price $p_j(s) = \frac{w_a C_{aj}}{w_b C_{bj}}$ of the second commodity is generated. The equilibrium price equalizes aggregate demand and aggregate supply at market j:

$$\begin{cases} x_{aj1}(p_j(s))C_{aj} + x_{bj1}(p_j(s))C_{bj} = (1-\phi_j)w_aC_{aj} \\ x_{aj2}(p_j(s))C_{aj} + x_{bj2}(p_j(s))C_{bj} = (1-\phi_j)w_bC_{bj}. \end{cases}$$

At the market in region i, every middleman obtains an equal share of the collected access fee in market i. This means, that the middlemen in region A consume a bundle

$$\left(\frac{\phi_A w_a C_{aA}}{M_A} + w_a - \sigma_A, \frac{\phi_A w_b C_{bA}}{M_A}\right) \text{ if } M_A > 0,$$

while the middlemen in region B consume a bundle

$$\left(\frac{\phi_B w_a C_{aB}}{M_B}, \frac{\phi_B w_b C_{bB}}{M_B} + w_b - \sigma_B\right) \text{ if } M_B > 0,$$

If $M_A = 0$ or $M_B = 0$, bundles $(w_a - \sigma_A, 0)$ and $(0, w_b - \sigma_B)$ are consumed, respectively; zero fractions of middlemen are assumed not to be able to set up a market.

A final element of the model is a *transportation cost*. Consumers of type $i \in \{a, b\}$ travelling from region i to region $j \in \{A, B\}$ to become a trader incur a transportation cost τ_{ij} in terms of the non-tradeable commodity, where $\tau_{aA} = \tau_{bB} = 0$, $0 < \tau_{aB} \leq L_a$, and $0 < \tau_{bA} \leq L_b$. Travelling to the market in one's own region is considered costless. The commodity could be leisure. This interpretation follows, e.g., Berliant and Wang (1993).

For state $s \in \Delta$, $f_i(s)$ is the payoff of a consumer becoming middlemen at region $i \in \{A, B\}$, and $g_{ij}(s)$ is the payoff of a type $i \in \{a, b\}$ consumer becoming trader at the market in region $j \in \{A, B\}$. The payoffs are equal to the utilities of the consumed bundles of commodities and leisure. Under the assumption that a zero fraction of middlemen cannot activate a market and a zero fraction of traders cannot trade, the payoffs of the middlemen in region A are

$$f_A(s) = \begin{cases} \left(\frac{\phi_A w_a C_{aA}}{M_A} + w_a - \sigma_A\right) \cdot \frac{\phi_A w_b C_{bA}}{M_A} + v(L_a) & \text{if } M_A > 0 \\ v(L_a) & \text{if } M_A = 0, \end{cases}$$

and the payoffs of the middlemen in region B are

$$f_B(s) = \begin{cases} \frac{\phi_B w_A C_{aB}}{M_B} \cdot \left(\frac{\phi_B w_B C_{bB}}{M_B} + w_b - \sigma_B\right) + v(L_b) & \text{if } M_B > 0 \\ v(L_b) & \text{if } M_B = 0. \end{cases}$$

For the consumers of type $i \in \{a, b\}$ going to the market in region $j \in \{A, B\}$ we find

$$g_{aA}(s) = \begin{cases} (1-\phi_A)^2 \frac{w_a w_b C_{bA}}{4C_{aA}} + v(L_a) & \text{if } M_A > 0 \text{ and } C_{aA} > 0 \\ v(L_a) & \text{otherwise,} \end{cases}$$

$$g_{aB}(s) = \begin{cases} (1-\phi_B)^2 \frac{w_a w_b C_{bB}}{4C_{aB}} + v(L_a - \tau_{aB}) & \text{if } M_B > 0 \text{ and } C_{aB} > 0 \\ v(L_a - \tau_{aB}) & \text{otherwise,} \end{cases}$$

$$g_{bA}(s) = \begin{cases} (1-\phi_A)^2 \frac{w_a w_b C_{aA}}{4C_{bA}} + v(L_b) & \text{if } M_A > 0 \text{ and } C_{bA} > 0 \\ v(L_b - \tau_{bA}) & \text{otherwise, and} \end{cases}$$

$$g_{bB}(s) = \begin{cases} (1-\phi_B)^2 \frac{w_a w_b C_{aB}}{4C_{bB}} + v(L_b) & \text{if } M_B > 0 \text{ and } C_{bB} > 0 \\ v(L_b) & \text{otherwise.} \end{cases}$$

Consumers are supposed to adopt roles (middleman or trader) and regions simultaneously, according to a Nash equilibrium. That is, roles and regions are adopted such that no consumer has an incentive to choose differently, given the choices of all other consumers. We refer to Definition 2.3.1 in Chapter 2.

3.3 Equilibria

This section examines the Nash equilibria of the game in which all consumers choose a role (middleman or trader) and a market simultaneously. Recalling condition (2.1) in

Chapter 2, a state is a (Nash) equilibrium if for each type, the payoffs of positive fractions are equal, and the payoffs of zero fractions do not exceed those of positive fractions. We distinguish the following kinds of equilibria.

Definition 3.3.1 *An equilibrium $s \in \Delta$ exhibits*

1. **No trade** *at A if $M_A = 0$ or $C_{aA} \cdot C_{bA} = 0$.*
2. **Concentrated trade** *at A if $M_A, C_{aA}, C_{bA} > 0$ and there is no trade at B.*
3. **Dispersed trade** *if $M_A, C_{aA}, C_{bA} > 0$ and $M_B, C_{aB}, C_{bB} > 0$.*

We call an equilibrium s a no trade equilibrium if neither A nor B have trade. A situation of no trade is encountered if either no market is activated, or only one type of trader is represented by a positive fraction. No trade equilibria are characterized as follows.

Proposition 3.3.2 *A non-empty collection Δ_0 of no-trade equilibria exists, where every equilibrium $s \in \Delta_0$ satisfies*

$$0 \leq M_A = 1 - C_{aA} \text{ and } 0 \leq M_B = 1 - C_{bB}.$$

PROOF

In any state s satisfying the condition we have $f_A(s) = g_{aA}(s) = v(L_a)$ and $f_B(s) = g_{bB}(s) = v(L_b)$. Moreover, $g_{aB}(s) = v(L_a - \tau_{aB}) < v(L_a)$ and $g_{bA}(s) = v(L_b - \tau_{bA}) < v(L_b)$, where $C_{aB} = C_{bA} = 0$. Hence, s is an equilibrium.

We cannot have a no-trade equilibrium with $M_A = 0$ and $C_{aA}, C_{bA} > 0$, for in that case, $g_{bA}(s) < v(L_b) = g_{bB}(s)$, which is a contradiction.

□

Equilibria have concentrated trade at a market if that market is active and both types are represented there as traders with a positive fraction, whereas the other market is not active. We derive the following collections of equilibria with concentrated trade. Two types are distinguished. In the first type, all consumers are middleman or trader at the active market. In the second type, a certain fraction of consumers goes to the inactive market, thereby consuming their initial endowment. We derive existence of equilibria with concentrated trade in region A. By symmetry, similar equilibria with concentrated trade in region B exist, which is not explicitly shown.

Proposition 3.3.3 *Define the following subspaces.*

$$\begin{aligned}\Delta_1(A) &:= \{s \in \Delta \mid 0 < M_A = 1 - C_{aA} < 1 \text{ and } C_{bA} = 1\}\\ \Delta_2(A) &:= \{s \in \Delta \mid 0 < M_A = 1 - C_{aA} < 1 \text{ and } 0 < C_{bA} < 1\}\end{aligned}$$

For every $\phi_A \in (0,1)$ there exists a bound $\bar{\tau}(\phi_A) > 0$ such that

- *If $\tau_{bA} \leq \bar{\tau}(\phi_A)$, there exists a unique equilibrium $s \in \Delta_1(A)$ exhibiting concentrated trade at market A.*
- *If $\tau_{bA} > \bar{\tau}(\phi_A)$, there exists a collection of equilibria in $\Delta_2(A)$ exhibiting concentrated trade at market A.*

PROOF

Define $\widetilde{w}_{aA} := w_a - \sigma_A$. Consider a state $s \in \Delta_1(A)$. The state s is a candidate equilibrium if $f_A(s) = g_{aA}(s)$, which is equivalent to

$$4w_a\phi_A^2 C_{aA}^2 + 4\widetilde{w}_{aA}\phi_A M_A C_{aA} - (1-\phi_A)^2 w_a M_A^2 = 0, \text{ or}$$

$$C_{aA} = \frac{\sqrt{\widetilde{w}_{aA}^2 + w_a^2(1-\phi_A)^2} - \widetilde{w}_{aA}}{2w_a\phi_A} M_A.$$

Together with the condition $M_A + C_{aA} = 1$ we obtain candidate equilibrium fractions

$$\hat{M}_A = \frac{2w_a\phi_A}{\sqrt{\widetilde{w}_{aA}^2 + w_a^2(1-\phi_A)^2} + 2w_a\phi_A - \widetilde{w}_{aA}} \text{ and}$$

$$\hat{C}_{aA} = \frac{\sqrt{\widetilde{w}_{aA}^2 + w_a^2(1-\phi_A)^2} - \widetilde{w}_{aA}}{\sqrt{\widetilde{w}_{aA}^2 + w_a^2(1-\phi_A)^2 + 2w_a\phi_A} - \widetilde{w}_{aA}}.$$

Since $f_A(\hat{s}) = g_{aA}(\hat{s}) > v(L_a) > v(L_a - \tau_{aB}) = g_{aB}(\hat{s})$, we require $g_{bA}(\hat{s}) \geq v(L_b) = f_{bB}(\hat{s})$ for $\hat{s}$ to be an equilibrium. This last condition is satisfied for all τ_{bA} smaller than or equal to some sufficiently small $\bar{\tau}(\phi_A)$, which exists by monotonicity and continuity of the function v.

Next, consider a state $\widetilde{s} \in \Delta_2(A)$. It is not difficult to see that we obtain candidate equilibrium fractions $\widetilde{M}_A = \hat{M}_A$ and $\widetilde{C}_{aA} = \hat{C}_{aA}$. The fraction $\widetilde{C}_{bA}$ is such that $g_{bA}(\widetilde{s}) = v(L_b)$, that is

$$\widetilde{C}_{bA} = (1-\phi_A)^2 \frac{w_a w_b \widetilde{C}_{aA}}{4(v(L_b) - v(L_b - \tau_{bA}))}.$$

The fraction $\widetilde{C}_{bA}$ exists whenever $\tau_{bA} > \bar{\tau}(\phi_A)$. The fractions $\widetilde{M}_B$ and $\widetilde{C}_{bB}$ may take any value as long as $\widetilde{M}_B + \widetilde{C}_{bB} = 1 - \widetilde{C}_{bA}$, in which case $f_B(\widetilde{s}) = g_{bB}(\widetilde{s}) = v_(L_b)$.

To show that there are no other equilibria with concentrated trade, consider the remaining possible states s with $C_{aB} > 0$. In s, we have payoffs $g_{aB}(s) = v(L_a - \tau_{aB}) < v(L_a)$, and $g_{aA}(s) \geq v(L_a)$, which is a contradiction.

□

The size of the transportation cost τ_{bA} determines whether all consumers are at the active market, or some consumers from region B, without the active market, 'stay home' and consume their initial endowments. If τ_{bA} is relatively large, part of the type b consumers does not enter market A as trader. Since these traders' payoff functions are decreasing in the fraction C_{bA}, the exclusion of some type b consumers from market A improves the traders' position. Consumers withdraw from the market until the transportation cost is traded off against gains-from-trade. In this case, only the type a traders have positive gains-from-trade, although both types are represented by a positive fraction.

In the following example, we illustrate the areas for access fee and transportation cost.

Example 3.3.4 We illlustrate the areas for the access fee and transportation cost for the limit case $\sigma_A = w_a$, that is, middlemen sacrifice their entire endowment. For the function v we take $v(l) = l - L_i$ for $i \in \{a, b\}$. Suppose the market in region A is active, then we have the following equilibrium fractions

$$\hat{M}_A = \frac{2\phi_A}{1+\phi_A},\ \hat{C}_{aA} = \frac{1-\phi_A}{1+\phi_A},\ \text{and}$$

$$\hat{C}_{bA} = \begin{cases} 1 & \text{if } \tau_{bA} \le \frac{w_a w_b (1-\phi_A)^3}{4(1+\phi_A)} \\ \frac{(1-\phi_A)^3 w_a w_b}{4\tau_{bA}(1+\phi_A)} & \text{otherwise.} \end{cases}$$

□

We derive two comparative statics results. The first one considers the influence of the access fee on the size of the equilibrium fraction of middlemen.

Corollary 3.3.5 *The equilibrium fraction of middlemen $\hat{M}_A$ is increasing in the access fee ϕ_A.*

PROOF

We have $\partial \hat{M}_A / \partial \phi_A > 0$ if

$$\sqrt{\widetilde{w}_{aA}^2 + w_a^2(1-\phi_A)^2 - \widetilde{w}_{aA}^2} - \widetilde{w}_{aA} > -\frac{\phi_A(1-\phi_A)w_a^2}{\sqrt{\widetilde{w}_{aA}^2 + w_a^2(1-\phi_A)^2} - \widetilde{w}_{aA}},$$

which condition holds, since $\sqrt{\widetilde{w}_{aA}^2 + w_a^2(1-\phi_A)^2} > \widetilde{w}_{aA}$.

□

The result is intuitive; a higher reward (access fee) for intermediation attracts more middlemen in equilibrium. Crucial however is the traders' payoff function. A higher access fee at market A increases the middlemen's payoffs, and decreases the traders' payoffs. The fact that a type a trader's payoff function is increasing in the fraction C_{aA}, and the payoff function of a type a middleman is decreasing in C_{aA}, requires a smaller equilibrium fraction of type a traders at market A, hence a larger middleman fraction. For general utility functions, a higher access fee would not necessarily decrease traders' payoffs. We could have the situation that payoffs increase by a higher access fee, because in the competitive equilibrium larger commodity bundles are obtained in spite of smaller net endowment bundles.

A second result is on the influence of the set-up cost on the middleman fraction.

Corollary 3.3.6 *The fraction $\hat{M}_A$ is decreasing in the set-up cost σ_A.*

If the set-up cost decreases, middlemen sacrifice less by entering the market, which makes it more attractive for them to enter. As an illustration, compare the extreme cases $\sigma_A = w_a$ and $\sigma_A = 0$. For $\phi_A = 1/4$, the equilibrium fraction of middlemen in the first case is $2/5$, and in the second case $2/3$.

Finally, we derive the existence of equilibria with dispersed trade. With dispersed trade, both markets are active and have both types represented as traders with a positive fraction. We find that a dispersed trade equilibrium exists for sufficiently large transportation costs.

Proposition 3.3.7 *There exist τ_a^*, τ_b^* such that for all $\tau_{aB} > \tau_a^*$ and $\tau_{bA} > \tau_b^*$, an equilibrium with dispersed trade s^* exists.*

Proof

We introduce the following short-hand notation for $i \in \{a, b\}$, and $j = A$ if $i = b$ and $j = B$ if $i = a$: $\alpha_i := (1 - \phi_i)^2 w_a w_b / 4$, and $\delta_i := v(L_i) - v(L_i - \tau_{ij})$.

The equilibrium equations $g_{aA}(s) = g_{aB}(s)$ and $g_{bA}(s) = g_{bB}(s)$ for traders yield the necessary conditions

$$\begin{cases} \frac{\alpha_A C_{bA}}{C_{aA}} = \frac{\alpha_B C_{bB}}{C_{aB}} - \delta_a \\ \frac{\alpha_A C_{aA}}{C_{bA}} = \frac{\alpha_B C_{aB}}{C_{bB}} + \delta_b, \end{cases} \tag{3.1}$$

Define the ratios $x = \frac{C_{bA}}{C_{aA}}$ and $y = \frac{C_{aB}}{C_{bB}}$. The system (3.1) yields the equilibrium ratios

$$x^*(\delta_a, \delta_b) = \frac{\alpha_A^2 - \alpha_B^2 - \delta_a\delta_b + \sqrt{(\alpha_A^2 - \alpha_B^2 - \delta_a\delta_b)^2 + 4\delta_a\delta_b\alpha_A^2}}{2\alpha_A\delta_B}$$

and $y^*(\delta_a, \delta_b) = \frac{\alpha_B}{\alpha_A x^*(\delta_a,\delta_b)+\delta_a}$.

The equilibrium fractions are found by solving

$$\begin{cases} \frac{\phi_A^2(w_aC_{aA}+w_a-\sigma_A)w_bC_{aA}}{(1-C_{aA}-y^*(\delta_a,\delta_bC_{bB})^2} = \alpha_A w_a w_b x^*(\delta_a, \delta_b) \\ \\ \frac{\phi_B^2 w_a C_{bB}(w_bC_{bB}+w_b-\sigma_B)}{(1-C_{bB}-x^*(\delta_a,\delta_b)C_{aA})^2} = \alpha_B w_a w_b y^*(\delta_a, \delta_b), \end{cases}$$

which yields the equations

$$\begin{cases} C_{aA} = \frac{1}{x^*(\delta_a,\delta_b)}\left(1 - C_{bB} - \sqrt{\frac{(\phi_B)^2 w_a C_{bB}(w_bC_{bB}+\widetilde{w}_b)}{\alpha_B w_a w_b y^*(\delta_a,\delta_b)}}\right) \\ C_{bB} = \frac{1}{y^*(\delta_a,\delta_b)}\left(1 - C_{aA} - \sqrt{\frac{(\phi_A)^2(w_aC_{aA}+\widetilde{w}_a)w_bC_{aA}}{\alpha_A w_a w_b x^*(\delta_a,\delta_b)}}\right). \end{cases} \tag{3.2}$$

The system (3.2) determines functions $C_{aA}(C_{bB})$ and $C_{bB}(C_{aA})$. We have $C_{aA}(0) = \frac{1}{x^*(\delta_a,\delta_b)} > 1$, and for some $0 < \bar{C}_{bB} < 1$: $C_{aA}(\bar{C}_{bB}) = 0$. Analogously, $C_{bB}(0) = \frac{1}{y^*(\delta_a,\delta_b)} > 1$, and for some $0 < \bar{C}_{aA} < 1$: $C_{bB}(\bar{C}_{aA}) = 0$. Together with the fact that the functions C_{aA} and C_{bB} are continuous and strictly decreasing in C_{bB} and C_{aA}, respectively, this means that there exists a unique pair (C_{aA}^*, C_{bB}^*) solving (3.2). The other fractions follow by substitution.

We have an equilibrium if $C_{aA}^* + y^*(\delta_a, \delta_b)C_{bB}^* < 1$ and $C_{bB}^* + x^*(\delta_a, \delta_b)C_{aA}^* < 1$. These conditions are satisfied for sufficiently large δ_a and δ_b, hence, for sufficiently large τ_{aB} and τ_{bA}, since $\lim_{\delta_a\to\infty}\lim_{\delta_b\to\infty} x^*(\delta_a, \delta_b) = 0$ and $\lim_{\delta_a\to\infty}\lim_{\delta_b\to\infty} y^*(\delta_a, \delta_b) = \infty$.

□

An illustration is provided in the following example.

Example 3.3.8 Consider the symmetric case $w_a = w_b = 2$, $v(l) = l - L_i$ for $i \in \{a, b\}$, $\phi_A = \phi_B =: \phi$, $\tau_{aB} = \tau_{bA} =: \tau$, and $\sigma_A = \sigma_B = 2$. Then, we find a bicentric equilibrium s^* with fractions

$$\begin{cases} M_A^* = \frac{2\phi}{2\phi+(1+x)(1-\phi)} \\ C_{aA}^* = C_{bB}^* = \frac{1-\phi}{2\phi+(1+x)(1-\phi)} \\ C_{aB}^* = C_{bA}^* = \frac{x(1-\phi)}{2\phi+(1+x)(1-\phi)}, \end{cases}$$

with $x = \frac{\sqrt{\tau^2+4(w_a w_b)^2(1-\phi)^4}-\tau}{2(1-\phi)^2}$.

If the transportation cost parameter τ is increased, the equilibrium trader fractions C_{aA}^* and C_{bB}^* increase, whereas the fractions C_{aB}^* and C_{bA}^* decrease. If for a trader travelling to the market in a region other than the region where he lives becomes more costly, he has an increasing tendency to go to the market in his own region. Also, the equilibrium middleman fractions M_A^* and M_B^* increase. A higher transportation cost makes it less attractive to become a trader at all, in favor of becoming a middleman.

□

In Figure 3.1, all equilibria are given, and also displayed graphically. A solid dot indicates a positive fraction of middlemen, where columns correspond to regions. An open dot with an arrow to some region indicates a positive fraction of traders travelling to that region. Finally, a small dot indicates a zero fraction of middlemen, that is, an inactive market.

3.4 Strong equilibria

In the previous section, a considerable number of Nash equilibria has been derived. This multiplicity is explained by the lock-in effects inherent to powerless individual agents, namely individual agents do not have the ability to set up markets. Also, traders being members of zero fractions are not able to trade. Although the empty market seems more attractive because of a lower access fee, it is not profitable for individual consumers to go there. This section resolves this locking-in by considering potential deviations by coalitions rather than individuals. Although individual traders have no incentive to deviate to an empty market, coalitions consisting of different types might have this incentive.

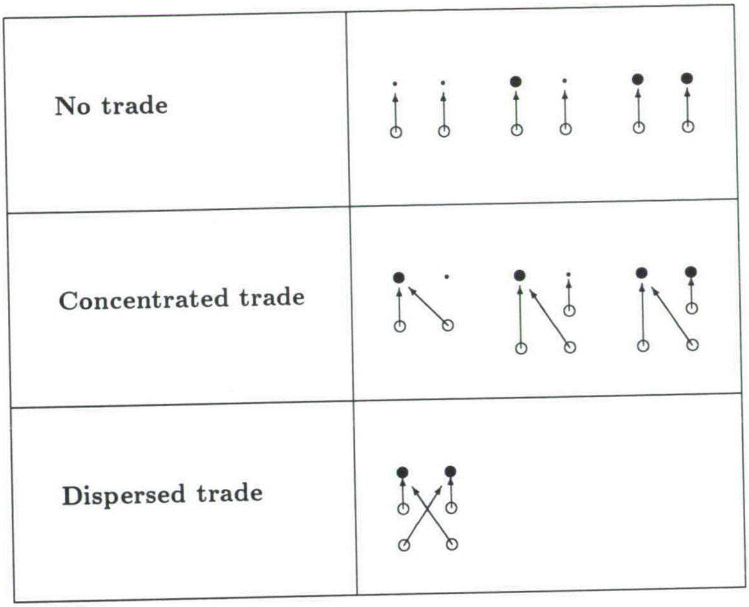

Figure 3.1: Overview of equilibria.

Coalitional stability of equilibria is captured by the concept of strong equilibrium. We refer to Definition 2.4.2 in Chapter 2.

In a strong equilibrium, the assignment of consumers to roles and markets is such that no coalition is able to find an assignment of roles and markets to its members that yields all its members a strict payoff improvement. In particular, the 'grand coalition' A cannot. Strong equilibria yield therefore an efficient division of payoffs, given the institutional structure of markets supporting the economy. Were roles and markets assigned by a social planner, strong equilibrium profiles would be candidate assignments.

Our first result concerns the equilibrium with concentrated trade in the collection $\Delta_1(A)$ (defined in Proposition 3.3.3), where all consumers are middleman or trader at the active market in region A.

Proposition 3.4.1 *Let $\hat{s}(A) \in \Delta_1(A)$ be an equilibrium with concentrated trade in region A. $\hat{s}(A)$ is a strong equilibrium if*

$$(((1-\phi_A)^4 - (1-\phi_B)^4)(w_a w_b)^2 - 16\delta_a\delta_b)\hat{C}_{aA} + 4\delta_a(1-\phi_A)^2 w_a w_b \hat{C}_{aA}^2 - 4(1-\phi_A)^2 w_a w_b \delta_b > 0 \tag{3.3}$$

with $\delta_a = v(L_a) - v(L_a - \tau_{aB})$ and $\delta_b = v(L_b) - v(L_b - \tau_{bA})$.

Proof

A strict coalitional improvement could be achieved by a coalition η of type b agents activating the market in region B as middlemen, and coalitions θ_a and θ_b of traders of type a and b, respectively, at market B. Let M be the size of coalition η and let C_a and C_b be the sizes of coalitions θ_a and θ_b, respectively.

The equilibrium s is strong if it is not possible for the coalition $\eta \cup \theta_a \cup \theta_b$ to deviate profitably. Since $\lim_{M_B \downarrow 0} f_B(s) = \infty$ on the space $\{s \in \Delta \mid M_B, C_{aB}, C_{bB} > 0\}$, it is always possible, given θ_a and θ_b, to find a coalition η of middlemen that strictly improves its members' payoffs by choosing M sufficiently close to zero. Therefore, we must consider deviations by traders.

A strict payoff improvement for all deviating traders from θ_a and θ_b is achieved if C_a, C_b exist such that

$$\begin{cases} (1-\phi_B)^2 w_a w_b \frac{C_b}{4C_a} - \delta_a > \frac{(1-\phi_A)^2 w_a w_b}{4\hat{C}_{aA}} \\ (1-\phi_B)^2 w_a w_b \frac{C_a}{4C_b} > (1-\phi_A)^2 w_a w_b \frac{\hat{C}_{aA}}{4} - \delta_b, \end{cases}$$

which is equivalent to

$$\begin{cases} \frac{C_b}{C_a} > \frac{(1-\phi_A)^2 w_a w_b + 4\delta_a \hat{C}_{aA}}{(1-\phi_B)^2 w_a w_b \hat{C}_{aA}} \\ \frac{C_b}{C_a} < \frac{(1-\phi_B)^2 w_a w_b}{(1-\phi_A)^2 w_a w_b \hat{C}_{aA} - 4\delta_b}. \end{cases}$$

These inequalities fail to hold, in which case $\hat{s}$ is strong, if

$$\frac{(1-\phi_B)^2 w_a w_b}{(1-\phi_A)^2 w_a w_b \hat{C}_{aA} - 4\delta_b} \leq \frac{(1-\phi_A)^2 w_a w_b + 4\delta_a \hat{C}_{aA}}{(1-\phi_B)^2 w_a w_b \hat{C}_{aA}},$$

which yields condition (3.3).

□

We consider the influence of transportation costs and set-up costs on inequality (3.3).

Corollary 3.4.2 *The left hand side of (3.3) is decreasing in τ_{bA}.*

As τ_{bA} increases, it becomes less attractive for type b consumers to become trader at region A. They have a stronger tendency to activate and use the market in their own region, and incite type a consumers to become trader at region B. In order to make it profitable for type a consumers to travel to region B, they should be compensated for incurred transportation costs τ_{aB}. Compensation is provided to a successfully deviating coalition in the form of increased gains-from-trade. Namely, in such a deviating coalition, the fraction of type a traders is small relative to the fraction of type b traders. Increased gains-from-trade for type a traders imply smaller gains for type b traders. They are compensated by not incurring a transportation cost τ_{bA}, however.

Corollary 3.4.3 *The left hand side of (3.3) is increasing in the set-up cost σ_A of market A.*

For larger set-up costs we have an increase in the degree of coalitional stability of the equilibrium $\hat{s}$. If the set-up cost of market A increases, there is less incentive for consumers to deviate to market B as a middleman.

Next, consider strongness of equilibria with dispersed trade. We derive that any equilibrium with dispersed trade is strong.

Proposition 3.4.4 *Any equilibrium s^* with dispersed trade is strong.*

PROOF

The equilibrium s^* satisfies

$$\begin{cases} (1-\phi_A)^2 \frac{w_a w_b C^*_{bA}}{4C^*_{aA}} + v(L_a) = (1-\phi_B)^2 \frac{w_a w_b C^*_{bB}}{4C^*_{aB}} + v(L_a - \tau_{aB}) \\ (1-\phi_A)^2 \frac{w_a w_b C^*_{aA}}{4C^*_{bA}} + v(L_b - \tau_{bA}) = (1-\phi_B)^2 \frac{w_a w_b C^*_{aB}}{4C^*_{bB}} + v(L_b). \end{cases} \tag{3.4}$$

Suppose coalitions of traders η_a and η_b of sizes $C_a \leq C^*_{aA}$ and $C_b \leq C^*_{bA}$, respectively, deviate from market A to market B. The members of these coalitions strictly increase their payoffs if

$$\begin{cases} (1-\phi_B)^2 w_a w_b \frac{C^*_{bB}+C_b}{4(C^*_{aB}+C_a)} + v(L_a - \tau_{aB}) > (1-\phi_A)^2 w_a w_b \frac{C^*_{bA}}{4C^*_{aA}} + v(L_a) \\ \\ (1-\phi_B)^2 w_a w_b \frac{C^*_{aB}+C_{aB}}{4(C^*_{bB}+C_b)} + v(L_b) > (1-\phi_A)^2 w_a w_b \frac{C^*_{aB}}{4C^*_{bB}} - v(L_b - \tau_{bA}). \end{cases} \tag{3.5}$$

By (3.4), the first inequality of (3.5) is satisfied only if

$$\frac{C^*_{bB}+C_b}{C^*_{aB}+C_a} > \frac{C^*_{bB}}{C^*_{aB}}.$$

However, this implies that

$$\frac{C^*_{aB}+C_a}{C^*_{bB}+C_b} < \frac{C^*_{aB}}{C^*_{bB}},$$

so that the second inequality of (3.5) is not satisfied. For the other cases it follows analogously that the two inequalities cannot be satisfied simultaneously. Hence, there does not exist a coalition strictly improving its members payoffs, so that s^* is a strong equilibrium.

□

Finally, we show that there are no other strong equilibria.

Proposition 3.4.5 *Let $s \in \Delta$ be a strong equilibrium. Then, either s is an equilibrium with concentrated trade in A or B, or s is an equilibrium with dispersed trade.*

PROOF

Consider coalitions of traders of sizes C_a and C_b and middlemen of type b of size M, deviating from market A to B.

Suppose first s is a no-trade equilibrium. In s, every type $i \in \{a, b\}$ consumer has a payoff $v(L_i)$. Consider a coalition consisting of positively measured groups of type a and b consumers becoming trader at market A, and of a positively measured group of type a consumers becoming middleman at market A. Let $C_a > 0$ and $C_b > 0$ be the sizes of the groups of traders of type a and b, respectively, and $M_A > 0$ the size of the group of middlemen.

The payoffs of the deviating type a consumers are strictly larger than $v(L_a)$ for any C_a, C_b and M_A. The type b traders strictly increase their payoff if

$$(1-\phi_A)^2 \frac{C_a}{4C_b} + v(L_b - \tau_{bA}) > v(L_b),$$

which condition is satisfied if the fraction C_a/C_b is sufficiently large. Therefore, s is not strong.

Suppose secondly that s is a state $\tilde{s}$ from the collection $\Delta_2(A)$ as defined in Proposition 3.3.3. Then, the type a consumers have a payoff

$$(1-\phi_A)^2 \frac{\widetilde{C}_{bA}}{4\widetilde{C}_{aA}} + v(L_a),$$

whereas the type b consumers have a payoff $v(L_b)$.

Consider a coalition consisting of positively measured groups of type a and b consumers becoming trader at market B, and of a positively measured group of type b consumers becoming middleman at market B. Let $C_a > 0$ and $C_b > 0$ be the sizes of the groups of traders of type a and b, respectively, and $M_B > 0$ the size of the group of middlemen.

The payoffs of the deviating type a traders are strictly increased if

$$(1-\phi_B)^2 \frac{C_b}{4C_a} + v(L_a - \tau_{aB}) > (1-\phi_A)^2 \frac{\widetilde{C}_{bA}}{4\widetilde{C}_{aA}} + v(L_a),$$

which is satisfied if the fraction C_b/C_a is sufficiently large. The payoffs of the deviating type b consumers are increased strictly for any sizes $M_B, C_a, C_b > 0$. Therefore, $\tilde{s} \in \Delta_2(A)$ is not strong.

$\square$

Chapter 4

Evolutionary Formation of Markets

4.1 Introduction

In this chapter, the process of market formation described by the static market formation game is analyzed in a dynamic framework in this chapter. Chapters 2 and 3 did not provide a real explanation of how consumers selected markets and roles, except for assuming perfect individual rationality leading to a Nash-equilibrium, or even coalitional rationality leading to a strong equilibrium. Here, we present an explicit model of the process of market formation by middlemen and market selection by traders. To be specific, we analyze the market formation game in an *evolutionary* framework. The idea behind this is as follows. Suppose the market formation game is played over and over again by the same population of consumers. At the start of every new period, endowments and utility functions as well as the set of markets are the same. Choices of markets and roles in a certain period are made according to their 'relative performance' in the previous period. That is, markets and roles that yielded relatively large payoffs are selected more frequently than others.

Such a process of selection of relatively profitable markets and roles is described by an *evolutionary dynamic*. Friedman (1991) gives an overview of this kind of dynamics[1]. A specific evolutionary dynamic is the *replicator dynamic*, as introduced by Taylor and Jonker (1978). Under the replicator dynamic, the growth rate of a fraction of consumers

[1] For an overview of evolutionary theory, we refer also to van Damme (1991,94), and Weibull (1995).

selecting a certain role and market is equal to the difference between that period's payoff associated with that role and market and the weighted average payoff of all consumers of the same type in that period. The long run outcomes of the replicator dynamic are the so-called *asymptotically stable states*, these are distributions that are reached with positive probability if time goes to infinity. That is, an asymptotically stable state has a positively measures 'basin of attraction', consisting of all distributions of consumers over roles and markets converging to the asymptotically stable state.

The dynamic asymptotically stable state has a static equivalent in the form of the *evolutionary stable state*. Bomze and Pötscher (1991) discuss analogies between static and dynamic evolutionary stability[2]. In particular, they show that evolutionary stability implies asymptotic stability under the replicator dynamic. The intuition behind this equivalence is that 'small' coalitions of deviating players, that is players not playing Nash equilibrium actions, obtain strictly lower payoffs amongst themselves than the Nash equilibrium players against the deviating players. Under the replicator dynamic, such deviating coalitions tend to disappear in the long run; the distributions 'perturbing' a certain asymptotically stable (evolutionary stable) state are in its basin of attraction. The equivalence between evolutionary and asymptotic stability holds if the state space is differentiable.

This chapter is two-fold. In the first part, we derive static evolutionary stability properties for the market formation model of Chapter 2. The second part considers dynamic properties under the replicator dynamic of a model similar to that of Chapter 3. The model of Chapter 2 describes market formation in an economy with a finite number of types and a countable collection of markets. Markets are activated by middlemen, who have free access to each market. The existence of two kinds of market equilibria is derived, in which several markets with sufficiently small access fees are activated. The first kind applies to a situation where markets have relatively small set-up costs, the second kind where set-up costs are not necessarily small. We apply the static evolutionary stability concept to these equilibria. It is shown that equilibria of the first kind are evolutionary stable, whereas equilibria of the second kind are evolutionary stable if the access fee of at least one active market is sufficiently small.

As mentioned already, evolutionary stability is a sufficient condition for asymptotic stability under the replicator dynamic if payoff functions are differentiable on the state space.

[2]See also Bomze and van Damme (1992).

In the market formation model of Chapter 2, this is not the case; a payoff discontinuity for zero fractions of middlemen and consumers exists. We propose a 'smoothing' procedure by assuming that 'negligible', i.e., positive but very small, fractions obtain nearly the same payoffs as zero fractions. With respect to middlemen, we may argue that small fractions are not able to activate a market properly; the small group of middleman causes considerable inefficiencies. With respect to consumers, we could have that negligible groups are not 'recognized' as trade partners.

If the state space is not 'smoothed' on the boundaries, evolutionary stability does no longer guarantee asymptotic stability. In the second part of this chapter, we analyze asymptotic stability of equilibria if the state space is discontinuous on the boundaries. We analyze a model with two types and two markets, similar to the model of Chapter 3, to which the replicator dynamic is applied. In this model, we have two types and two markets. The model has a spatial dimension; types and markets are associated with regions, between which transportation costs exist. Each market is located in one region, and each type inhabites one region. Middlemen are only able to activate the market in the region in which they live.

The market formation model of the second part yields two kinds of asymptotically stable states. In the first kind, all consumers select the same market, either as a middleman or a trader. Such a distribution $\hat{s}$, is asymptotically stable if the access fee of the market, say market i, adopted by all consumers is sufficiently small with respect to the transportation cost of the type $j \neq i \in I$ consumers to market i. The intuition behind this result is as follows. In $\hat{s}$, only the type j consumers inter transportation costs actually, by having to travel to market i. This means that type j consumers have a tendency to deviate to the market in their own region, in order to forego the transportation costs. If this tendency is strong enough, certain deviations by small coalitions of consumers to market j in the form of one-shot perturbations of $\hat{s}$ destabilize $\hat{s}$ in the long run, in the sense that the distribution never returns to $\hat{s}$. In order to maintain asymptotic stability, the transportation cost incurred by type j consumers must be traded off by the access fee of market i. In the second type of asymptotically stable state, the consumers disperse themselves over both markets.

An important feature of the dynamic model in the second part is that the economy is not changed over time; both individual (endowments and utility functions) and institutional characteristics (markets) are identical in every period. At the end of a period, consumers consume the bundles they leave the market with, and start 'from scratch' in the next period with new, time-independent endowments. We could imagine that endowments are time-dependent endowments, though. Parts of traded bundles could be transferred to next periods' endowments instead of consumed entirely in one period. In that case, we would incorporate effects of wealth accumulation. A reason for the absence of wealth accumulation could be that commodities are perishable. With respect to markets, we could imagine that markets change because of innovations. For instance, the set-up costs of markets decrease. A real-life example of decreasing set-up costs is provided by the financial intermediation sector. More and more, traditional banks with buildings and counters are replaced by share investment companies who have only a mailing address.

The replicator dynamic is applied to matrix games with bilateral matching, mainly. Typically, in these models with two populations, every agent is matched randomly with another agent from the other population after which a matrix game is played by all agents. In the bi-population case, the payoff of every type from population $i \in \{1,2\}$ is therefore linearly depending on the composition of types in population $j \neq i \in \{1,2\}$. Our application of the replicator dynamic differs from matrix games with bilateral matching in the sense that payoff functions are depending non-linearly on the distributions of types over markets and roles. A trader's payoff function is influenced negatively by the fraction of traders of his own type at his market. On the other hand, it is influenced positively by the fraction of traders of the other type. This reflects that the consumers seek *heterogeniety*: They desire to meet consumers of the other type, carrying the commodity they do not possess initially, whereas they want to avoid their own type.

This chapter is organized as follows. In Section 2, evolutionary stability properties of the model of Chapter 2 are examined. Section 3 investigates the asymptotically stable states of a two types, two markets model similar to Chapter 3 under the replicator dynamic. In Section 4, the actual evolution of states is considered. Proofs are gathered in Section 5.

4.2 Evolutionary Stability of market equilibria

In this section, we examine evolutionary stability properties of the market formation model formulated in Chapter 2. We will repeat the model shortly.

An *economy* E consists of an economic primitive E_0 (satisfying the assumptions of Definition 2.2.1) and a collection of markets Γ. The primitive E_0 is a tuple $(A^i, u^i, w^i)_{i\in N}$, where A^1 to A^n are coalitions of consumers of measure $1/n$, $(N = \{1, \ldots, n\})$, $w^i \in X = \mathbb{R}^l_+$ is the endowment of a type $i \in N$ consumer, and $u^i : X \to \mathbb{R}$ is his utility function. The economic primitive satisfies the conditions of Definition 2.2.1. To every market $\gamma \in \Gamma$ a market characteristic $\psi(\gamma) = (\sigma, \phi) \in X^{2n}$ consisting of a set-up cost $\sigma = (\sigma^i_1, \ldots, \sigma^i_l)_{i\in N} \in X^n$ and an *access fee* $\phi = (\phi^i_1, \ldots, \phi^i_l)_{i\in N} \in X^n$, with $\sigma^i_k \leq w^i_k$ and $\phi^i_k \leq w^i_k$ for all $i \in N$ and $k \in L$.

Markets are activated by middlemen and used by traders. On entering market $\gamma \in \Gamma$, a middleman pays its set-up cost, whereas a trader pays its access fee to the middlemen. Traders leave an active market with a competitive equilibrium bundle resulting from the endowments reduced by access fees of all traders at that market. They have a continuous belief over equilibrium bundles.

The choices of roles $r \in \{m, c\}$ (middleman or trader) and markets $\gamma \in \Gamma$ by consumers of type $i \in N$ are supposed to yield measurable groups $A^i_{\gamma r}$. A distribution of agents over markets and roles is a tuple $s = (s^1, s^2, \ldots, s^n)$, with for every $i \in N$: $s^i = (s^i_{\gamma m}, s^i_{\gamma c})_{\gamma\in\Gamma}$.

Here, $s^i_{\gamma m}$ is the fraction of middlemen of type $i \in N$ at market $\gamma \in \Gamma$, and $s^i_{\gamma c}$ is the fraction of traders of type i at γ.

Markets γ and roles $r \in \{m, c\}$ are chosen simultaneously in the market formation game, yielding type i a payoff $f^i_{\gamma r}(s)$. In Lemma 2.2.13, continuity of payoff functions is derived. In this chapter, we make an additional assumption of differentiability.

Assumption 4.2.1 *For any market $\gamma \in \Gamma$ and type $i \in N$, the payoff function $f^i_{\gamma r}$ is differentiable on the subspace*

$$\{s \in \Delta \mid \sum_{j\in N} s^j_{\gamma m} > 0 \text{ and } s^i_{\gamma c} > 0\}.$$

An evolutionary stable state for the market formation model is defined as follows (cf. Friedman (1991)).

Definition 4.2.2 *For $i \in N$ and $s \in \Delta$, let $f^i(s) = (f^i_{\gamma r}(s))_{\gamma \in \Gamma, r \in \{m,c\}}$.*

A state $s^* \in \Delta$ is an **Evolutionary Stable State** *if for each $i \in N$ and $s \in \Delta \setminus \{s^*\}$ one of the following conditions holds.*

1. $s^i \cdot f^i(s^*) < s^{i*} \cdot f^i(s^*)$
2. $s^i \cdot f^i(s^*) = s^{*i} \cdot f^i(s^*)$ *and* $s^i \cdot f^i(s^*_{|i}(\epsilon)) < s^{*i} \cdot f^i(s^*_{|i}(\epsilon))$ *for some* $\epsilon > 0$, *where*

   $$s^*_{|i}(\epsilon) := (s^{*1}, \ldots, s^{*,i-1}, (1-\epsilon)s^{*i} + \epsilon s^i, s^{*,i+1}, \ldots, s^{*n})$$

   is a perturbation of s^{*i} towards s^i.

The first part of the definition is the Nash-equilibrium condition. In a Nash equilibrium, no agent has an incentive to select a different role or market, given the choices of other consumers. Condition 2 reflects stability. This condition selects the 'stable' equilibria among the different Nash equilibria, by considering the effect of the influx of small coalitions of consumers choosing different roles and markets. The intuition behind this selection criterion is as follows. Suppose that the market formation game is played repeatedly for many times. In every new play of the game, consumers face identical conditions. Instead of assuming that the same set of consumers plays the game repeatedly, one could also assume that every new game is played by an entirely new 'generation'. The individual characteristics of the consumers of each generation are 'inherited' by the consumers from the next generation. Also, the characteristics of markets remain constant in each generation. The agents of a new generation make their decisions based on the relative payoffs of the different roles obtained by former generations. 'Better' roles and markets are chosen more often than less performing ones.

The dynamical interpretation of evolutionary stability given above holds only if the payoff functions are differentiable on the entire state space. The market formation game has the property, that payoff functions are discontinuous for zero fractions of middlemen and traders. We propose a 'smoothing' procedure by assuming that 'negligible', i.e., positive but very small, fractions obtain nearly the same payoffs are zero fractions. This is reflected in the property of of evolutionary stable states that ϵ-deviations do not matter much if ϵ is small. For middlemen, we could assume that that 'small' coalitions are not able to collect efficiently the access fees, and to redistribute the resulting Walrasian bundles to the traders. So, *transaction costs* play a role. For consumer fractions, we have that consumers from 'small' fractions do not obtain their entire Walrasian bundle. An explanation could again be given in terms of transaction costs, namely that small consumer fractions are relatively costly for the middlemen to serve.

Theorem 4.2.3 *Let be given numbers $\delta > 0$ and $\epsilon > 0$, and a subset $\Gamma^* \subset \Gamma$ with $\#\Gamma^* \leq n/2$ such that for every $\gamma \in \Gamma^*$ its market characteristic $\psi(\gamma) = (\sigma, \phi)$ satisfies*

1. *$||\phi^i|| < \delta$ for all $i \in N$.*
2. *$||\sigma^i|| < \epsilon$ for all $i \in N$.*

If δ and ϵ are sufficiently small, then there generically exists an evolutionary stable market equilibrium $\hat{s} \in \Delta$ such that all markets $\gamma \in \Gamma^*$ are active.

For the proof, we refer to Section 4.5. A negative definiteness argument is applied to the matrix of partial derivatives of the payoff functions. The structure of the equilibrium is identical to the one derived in Theorem 2.3.6. More specifically, if a state is an equilibrium of this kind, it is also evolutionary stable.

We illustrate the theorem in the following example.

Example 4.2.4 Let be given an economy with 5 types and 2 commodities. Type 1 agents have a utility function $u^1(x_1, x_2) = x_1 + x_2$ and an endowment $(\alpha, \beta) \in \mathbb{R}^2_+$. Type 2 and 4 agents have a utility function $u^2(x_1, x_2) = x_2$ and an endowment (1,0). Finally, type 3 and 5 agents have a utility function $u^3(x_1, x_2) = x_1$ and an endowment (0,1).

Suppose markets γ_1 and γ_2 are active with access fees in the form of proportions of endowments $\phi_1 \in [0, 1/2)$ and $\phi_2 \in [0, 1/2)$, respectively. The set-up costs of both markets are zero.

We consider a market equilibrium in which type 1 agents disperse as middlemen over both markets, type 2 and 3 agents are consumers at market γ_1, and type 4 and 5 agents are consumers at market γ_2.

Let M_1 and M_2 denote the fractions of type 1 middlemen at market γ_1 and γ_2, respectively, and $C_{ij} \in [0,1]$ the fraction of consumers of type i at market γ_j. Middlemen's payoffs on the space

$$\Delta^* = \{s \in \Delta \mid 0 < M_1 = 1 - M_2 \text{ and } C_{11} = C_{21} = C_{32} = C_{42} = 1\}$$

are $f_1(s) = 2\phi_1/M_1$ and $f_2(s) = 2\phi_2/M_2$ for the middlemen at market γ_1 and γ_2, respectively. Consumers' payoffs at market γ_i, $i \in \{1,2\}$, are $2(1-\phi_i)$.

In a market equilibrium $\hat{s} \in \Delta^*$ we have $f_1(\hat{s}) = f_2(\hat{s})$, implying $\hat{M}_1 = \frac{\phi_1}{\phi_1+\phi_2}$ and $\hat{M}_2 = \frac{\phi_2}{\phi_1+\phi_2}$. Since access fees are strictly smaller than $1/2$, consumers have a strict incentive not to become a middleman at either market.

On the space Δ^* we have $M_1 f_1(\hat{s}) + M_2 f_2(\hat{s}) = \hat{M}_1 f_1(\hat{s}) + \hat{M}_2 f_2(\hat{s})$ for every $0 < M_1, M_2 < 1$. Therefore, by the second condition of Definition 4.2.2, evolutionary stability of $\hat{s}$ is satisfied if

$$M_1 \frac{2\phi_1}{(1-\epsilon)\hat{M}_1 + \epsilon M_1} + M_2 \frac{2\phi_2}{(1-\epsilon)\hat{M}_2 + \epsilon M_2} <$$

$$\hat{M}_1 \frac{2\phi_1}{(1-\epsilon)\hat{M}_1 + \epsilon M_1} + \hat{M}_2 \frac{2\phi_2}{(1-\epsilon)\hat{M}_2 + \epsilon M_2} \text{ for all } 0 < M_1 = 1 - M_2.$$

This condition boils down to

$$(\phi_1 + \phi_2)^2 M_1^2 - 2(\phi_1^2 + \phi_1\phi_2) M_1 + \phi_1^2 > 0,$$

which is satisfied for all $M_1 \neq \hat{M}_1$, since the left hand side becomes minimal for $M_1 = \hat{M}_1$ with minimum zero.

□

Theorem 4.2.5 *Let be given numbers $\delta > 0$ and $\epsilon > 0$, a type $k \in N$, and a non-empty subset $\Gamma^* \subset \Gamma$ with $\#\Gamma^* \leq n/2$ such that for all $\gamma \in \Gamma^*$ their market characteristics $\psi(\gamma) = (\sigma, \phi)$, and endowments w^i, $i \in N \setminus \{k\}$ have the property that*

1. $||\phi^i|| < \delta$ *for all* $i \in N$.
2. $\frac{||w^k||}{||w^i||} < \epsilon$.

If δ and ϵ are sufficiently small, then there generically exists an evolutionary stable market equilibrium $\hat{s} \in \Delta$ with the property that all markets $\gamma \in \Gamma^*$ are active.

For the proof, we refer to Section 4.5. Like in the proof of Theorem 4.2.3, a negative definiteness argument is applied to the matrix of partial derivatives of the payoff functions. The structure of the equilibrium has the same structure as in Theorem 2.3.9. The structure of the equilibrium is identical to the one derived in Theorem 2.3.9. It is not true, however, that every equilibrium derived there is also evolutionary stable, as in Theorem 4.2.3. The reason is that in the proof of Theorem 4.2.3, only the middlemen's payoff functions matter in the negative definiteness argument. These functions are monotonically decreasing in the sizes of the middlemen's fractions, which makes that any market equilibrium implies negative definiteness. In the proof of Theorem 4.2.5, also a consumer's payoff function matters, which does not satisfy monotonicity in the size of the consumer's fraction, in general.

4.3 Evolutionary selection under the replicator dynamic

This section is devoted to dynamical properties of a two region, two types economy model similar to the model of Chapter 3, by application of the two-population replicator dynamic (Taylor and Jonker (1978)).

Consider an economy with two spatially separated regions A and B. Region A is inhabited by a continuum of type a consumers, and region B by a continuum of type b consumers. Both types have equal Lebesgue measure. There are two commodities. Type

a and b consumers have initial commodity endowments $(1,0)$ and $(0,1)$, respectively. The consumers' utility functions are

$$u^a(x_1,x_2) = x_2 \text{ and } u^b(x_1,x_2) = x_1,$$

where $x_i \geq 0$ is the quantity of commodity $i \in \{1,2\}$.

The traders may trade their commodity endowments at one of two markets, each of which is competitive. After all consumers have selected a market, a competitive equilibrium is determined on both markets. The equilibria are local, in the sense that the equilibrium at market i depends on the composition of trader types at market i only. If a type a consumer becomes trader at the market in region A, this is costless for him in terms of travel time. On the other hand, if he goes to the market in region B, he incurs a transportation cost $\tau_{aB} \geq 0$ in terms of time. Analogously, if a type b consumer becomes a trader at the market in region A, he incurs a transportation cost $\tau_{bA} \geq 0$. Transportation costs enter the payoff function by subtracting them from the utility of consumed commodities. This specification is a special case of the separable form used in Chapter 3.

In the competitive equilibrium at market $j \in \{A,B\}$, type a traders obtain a commodity bundle $(0, \frac{C_{bj}}{C_{aj}})$ if $C_{aj} > 0$. If $C_{aj} = 0$, the traders return with their endowment bundle Type b traders obtain a bundle $(\frac{C_{aj}}{C_{bj}}, 0)$ if $C_{bj} > 0$, and their endowments otherwise. Type i's gains-trom-trade is thus inversely proportional to the size of its own traders' fraction.

The payoff $f_A(s)$ of a middlemen from region A is

$$f_A(s) = \begin{cases} \frac{\phi_A C_{bA}}{M_A} & \text{if } M_A > 0 \\ \\ 0 & \text{if } M_A = 0. \end{cases},$$

and of a middlemen from region B

$$f_B(s) = \begin{cases} \frac{\phi_B C_{aB}}{M_B} & \text{if } M_B > 0 \\ \\ 0 & \text{if } M_B = 0. \end{cases}$$

Only middleman fractions of positive size are able to activate a market. Middlemen from a group of zero measure, not being able to collect access fees, have the utility of their reduced endowment only.

The payoffs $g_{ij}(s)$ of traders of type $i \in \{a, b\}$ going to region $j \in \{A, B\}$ are as follows.

$$g_{aA}(s) = \begin{cases} (1-\phi_A)\frac{C_{bA}}{C_{aA}} & \text{if } M_A > 0 \text{ and } C_{aA} > 0 \\ 0 & \text{otherwise,} \end{cases}$$

$$g_{aB}(s) = \begin{cases} (1-\phi_B)\frac{C_{bB}}{C_{aB}} - \tau_{aB} & \text{if } M_B > 0 \text{ and } C_{aB} > 0 \\ -\tau_{aB} & \text{otherwise,} \end{cases}$$

$$g_{bA}(s) = \begin{cases} (1-\phi_A)\frac{C_{aA}}{C_{bA}} - \tau_{bA} & \text{if } M_A > 0 \text{ and } C_{bA} > 0 \\ -\tau_{bA} & \text{otherwise, and} \end{cases}$$

$$g_{bB}(s) = \begin{cases} (1-\phi_B)\frac{C_{aB}}{C_{bB}} & \text{if } M_B > 0 \text{ and } C_{bB} > 0 \\ 0 & \text{otherwise.} \end{cases}$$

Just like middlemen, zero fractions of consumers at an active market obtain the utility of their reduced endowment.

We consider the evolution of states under the continuous time two-population replicator dynamic. Therefore, in the sequel states are time-dependent: $s = s(t)$ for $t \geq 0$. For convenience, the reference to time will be deleted. Under the replicator dynamic, the growth rate of each fraction is equal to the difference between its members' payoffs and the weighted average payoff of all consumers of their type. We have the following system of differential equations for the traders, where a dot indicates the derivative with respect to time:

$$\begin{cases} \dot{C}_{aA} = C_{aA}(g_{aA}(s) - \pi_a(s)) \\ \dot{C}_{aB} = C_{aB}(g_{aB}(s) - \pi_a(s)) \\ \dot{C}_{bA} = C_{bA}(g_{bA}(s) - \pi_b(s)) \\ \dot{C}_{bB} = C_{bB}(g_{bB}(s) - \pi_b(s)), \end{cases} \tag{4.1}$$

with $\pi_a(s) = M_A f_A(s) + C_{aA} g_{aA}(s) + C_{aB} g_{aB}(s)$ and $\pi_b(s) = M_B f_B(s) + C_{bA} g_{bA}(s) + C_{bB} g_{bB}(s)$ the weighted average payoffs. Since we have $\dot{M}_A = -\dot{C}_{aA} - \dot{C}_{aB}$ and $\dot{M}_B = -\dot{C}_{bA} - \dot{C}_{bB}$, we need not consider the evolution of the middleman fractions explicitly.

Weighted average payoffs are

$$\pi_a(s) = C_{bA} + (1 - \phi_B)C_{bB} - \tau_{aB}C_{aB}, \text{ and}$$

$$\pi_b(s) = C_{aB} + (1 - \phi_A)C_{aA} - \tau_{bA}C_{bA}.$$

Thus, we arrive at the following system of differential equations.

$$\begin{cases} \dot{C}_{aA} = (1 - \phi_A)C_{bA} - C_{aA}C_{bA} - (1 - \phi_B)C_{aA}C_{bB} + \tau_{aB}C_{aA}C_{aB} \\ \\ \dot{C}_{aB} = (1 - \phi_B)C_{bB} - \tau_{aB}C_{aB} - C_{aB}C_{bA} - (1 - \phi_B)C_{aB}C_{bB} + \tau_{aB}C_{aB}^2 \\ \\ \dot{C}_{bA} = (1 - \phi_A)C_{aA} - \tau_{bA}C_{bA} - C_{aB}C_{bA} - (1 - \phi_A)C_{bA}C_{aA} + \tau_{bA}C_{bA}^2 \\ \\ \dot{C}_{bB} = (1 - \phi_B)C_{aB} - C_{aB}C_{bB} - (1 - \phi_A)C_{aA}C_{bB} + \tau_{bA}C_{bA}C_{bB}. \end{cases}$$

The collection of states in which no consumer changes markets, is the collection of stationary states.

Definition 4.3.1 *A state $s \in \Delta$ is a* **stationary state** *if*

$$\dot{C}_{ij} = 0 \text{ for all } i \in \{a, b\} \text{ and } j \in \{A, B\}.$$

Lemma 4.3.2 *The set S of stationary states is given by*

$$S = \begin{cases} \{\hat{s}(A), \hat{s}(B), s^*\} & \text{if } \begin{cases} (1+\tau_{aB})((1-\phi_A)^2 - \tau_{bA}) < (1-\phi_B)^2 \\ (1+\tau_{bA})((1-\phi_B)^2 - \tau_{aB}) < (1-\phi_A)^2, \end{cases} \\ \\ \{\hat{s}(A), \hat{s}(B)\} & \text{otherwise,} \end{cases}$$

where $\hat{s}(A)$ satisfies $\hat{M}_A = \phi_A, \hat{C}_{aA} = 1 - \phi_A$, and $\hat{C}_{bA} = 1$, $\hat{s}(B)$ satisfies $\hat{M}_B = \phi_B, \hat{C}_{bB} = 1 - \phi_B$, and $\hat{C}_{aB} = 1$, and s^* is a unique state satisfying $M_i^*, C_{jk}^* > 0$ for all $i, k \in \{A, B\}$ and $j \in \{a, b\}$.

PROOF

Define the space $\Delta_A := \{s \in \Delta \mid M_A = 1 - C_{aA}\}$. On Δ_A, the equations $\dot{C}_{aA} = \dot{C}_{aB} = \dot{C}_{bA} = 0$ yield the unique stationary state $\hat{s}(A)$. Analogously, the unique stationary state $\hat{s}(B)$ is found on the space $\Delta_B := \{s \in \Delta \mid M_B = 1 - C_{bB}\}$.

On the space

$$\Delta^* := \{s \in \Delta \mid M_i^*, C_{jk}^* > 0 \text{ for all } i, k \in \{A, B\} \text{ and } j \in \{a, b\}\},$$

a stationary state can be found by solving

$$\begin{cases} f_A(s) = g_{aA}(s) = g_{aB}(s) \\ f_B(s) = g_{bA}(s) = g_{bB}(s). \end{cases}$$

We solve

$$\begin{cases} (1-\phi_A)\frac{C_{bA}}{C_{aA}} = (1-\phi_B)\frac{C_{bB}}{C_{aB}} - \tau_{aB} \\ (1-\phi_A)\frac{C_{aA}}{C_{bA}} = (1-\phi_B)\frac{C_{aB}}{C_{bB}} + \tau_{bA}. \end{cases}$$

Multiplying the two equations yields

$$C_{bA} = xC_{aA} \text{ and } C_{bB} = yC_{aB},$$

with

$$x = \frac{\alpha_A^2 - \alpha_B^2 - \tau_{aB}\tau_{bA} + \sqrt{(\alpha_B^2 - \alpha_A^2 - \tau_{aB}\tau_{bA})^2 + 4\tau_{aB}\tau_{bA}\alpha_B^2}}{2\tau_{bA}\alpha_A} \text{ and}$$

$$y = \frac{\alpha_A^2 - \alpha_B^2 + \tau_{aB}\tau_{bA} + \sqrt{(\alpha_B^2 - \alpha_A^2 + \tau_{aB}\tau_{bA})^2 + 4\tau_{aB}\tau_{bA}\alpha_B^2}}{2\tau_{bA}\alpha_B},$$

where $\alpha_A = 1 - \phi_A$ and $\alpha_B = 1 - \phi_B$.

From the equations $f_A(s) = g_{aA}(s)$ and $f_B(s) = f_{bB}(s)$ we consequently solve

$$C_{aA} = \tfrac{\alpha_A(y-\alpha_B)}{y-\alpha_A\alpha_B x},\ C_{aB} = \tfrac{\alpha_B(1-\alpha_A x)}{y-\alpha_A\alpha_B x},\ C_{bA} = \tfrac{\alpha_A x(y-\alpha_B)}{y-\alpha_A\alpha_B x},\ \text{ and } C_{bB} = \tfrac{\alpha_B y(1-\alpha_A x)}{y-\alpha_A\alpha_B x},$$

under the condition

$$\begin{cases} (1+\tau_{aB})((1-\phi_A)^2 - \tau_{bA}) < (1-\phi_B)^2 \\ \\ (1+\tau_{bA})((1-\phi_B)^2 - \tau_{aB}) < (1-\phi_A)^2, \end{cases}$$

guaranteeing $0 < C_{aA} + C_{aB} < 1$ and $0 < C_{bA} + C_{bB} < 1$.

□

It should be noted, that the collection of stationary states does not coincide with the set of Nash equilibria of the market formation game. This latter set is larger; the set

$$\{s \in \Delta \mid M_A = \phi_A, C_{aA} = 1 - \phi_A \text{ and } C_{bA} < 1\}$$

consists of equilibria if $\tau_{bA} > (1-\phi_A)^2$. This discrepancy is caused by the discontinuity of the payoff functions for zero fractions.

We are interested in the subcollection of stationary states which are asymptotically stable. A stationary state s^* is asymptotically stable if sufficiently small one-time random perturbations of s^* do not influence s^* in the long run. That is, if s^* is perturbed in a certain period, the state evolves back to s^* if time goes to infinity. Asymptotically stable states will be observed with positive probability, as opposed to unstable stationary states. These latter states tend to 'disappear' in the long run, since arbitrarily small one-time perturbations lead the state away from them.

Definition 4.3.3 *A stationary state s^* is* **asymptotically stable**, *if there exists an open neighborhood $U(s^*)$ of s^* such that*

$$s(0) \in U(s^*) \text{ implies } \lim_{t\to\infty} s(t) = s^*,$$

where $t \geq 0$ indicates time, and $s(0)$ is the initial state.

Our first result is on the stability of the stationary states $\hat{s}(A)$ and $\hat{s}(B)$.

Proposition 4.3.4

1. *The state $\hat{s}(A)$ is asymptotically stable if*

$$(1+\tau_{aB})((1-\phi_A)^2-\tau_{bA}) > (1-\phi_B)^2.$$

2. *The state $\hat{s}(B)$ is asymptotically stable if*

$$(1+\tau_{bA})((1-\phi_B)^2-\tau_{aB}) > (1-\phi_A)^2.$$

It is easily derived that stability of $\hat{s}(A)$ excludes stability of $\hat{s}(B)$, and vice versa. The proof of the proposition is based on an eigenvalue analysis and is found in Section 4.5.

Consider the state $\hat{s}(A)$. The degree of stability of $\hat{s}(A)$ is decreasing in ϕ_A and τ_{bA}, and increasing in ϕ_B and τ_{aB}, in the sense that stability is satisfied for less, respectively more parameters. An increase in ϕ_A makes type a traders inclined more to trade at market B instead, although transportation costs τ_{aB} will be incurred then. Type b traders are however willing to offer higher gains-from-trade to type a traders by decreasing their own gains-from-trade. If this increase compensates for the transportation cost, type a traders will go to market B. The same reasoning applies to the case that τ_{bA} increases; type b

traders have a stronger inclination to incite type a traders to go to region B by offering them better terms-of-trade. For ϕ_B and τ_{aB}, an opposite argument applies.

For a dispersed trade state, we are able to derive not only asymptotic stability locally, but also globally. Namely, we derive that any initial state on the interior of the state space

$$int(\Delta) = \{s \in \Delta \mid M_i, C_{ai}, C_{bi} > 0 \text{ for all } i \in \{A, B\}\}$$

converges to the dispersed trade state.

Definition 4.3.5 *An asymptotically stable state s^* is* **globally stable** *if*

$$s(0) \in int(\Delta) \text{ implies } \lim_{t\to\infty} s(t) = s^*.$$

Proposition 4.3.6 *The stationary state with dispersed trade s^* is globally stable.*

In order to proof the result, we derive first two lemma's. The first lemma is straightforward.

Lemma 4.3.7 *The payoff functions are differentiable on $int(\Delta)$.*

The second lemma follows from Cressman (1995, Theorem 3.4).

Lemma 4.3.8 *If a state $s^* \in int(\Delta)$ is an evolutionary stable state, and the payoff functions are differentiable on $int(\Delta)$, then s^* is globally stable under the replicator dynamic.*

Proof of Proposition 4.3.6

We show that s^* is evolutionary stable. Therefore, we derive that s^* is a *regular* evolutionary stable state, that is, the matrix of partial derivatives

$$D_A(s^*) = \begin{pmatrix} \frac{\partial f_A}{\partial M_A}(s^*) & \frac{\partial f_A}{\partial C_{aA}}(s^*) & \frac{\partial f_A}{\partial C_{aB}}(s^*) \\ \frac{\partial g_{aA}}{\partial M_A}(s^*) & \frac{\partial g_{aA}}{\partial C_{aA}}(s^*) & \frac{\partial g_{aA}}{\partial C_{aB}}(s^*) \\ \frac{\partial g_{aB}}{\partial M_A}(s^*) & \frac{\partial g_{aB}}{\partial C_{aA}}(s^*) & \frac{\partial g_{aB}}{\partial C_{aB}}(s^*) \end{pmatrix}$$

is negative definite on the tangent space $H_A(s^*) = \left\{z \in \mathbb{R}^3 \mid \sum_{i=1}^3 z_i = 0\right\}$ at s^*. Analogously, negative definiteness is shown for the matrix $D_B(s^*)$. Taylor and Jonker (1978) show that every regular evolutionary stable state is also evolutionary stable.

The matrix $D_A(s^*)$ is

$$\begin{pmatrix} -\frac{\phi_A C^*_{bA}}{(M^*_A)^2} & 0 & 0 \\ 0 & -\frac{(1-\phi_A)C^*_{bA}}{(C^*_{aA})^2} & 0 \\ 0 & 0 & -\frac{(1-\phi_B)C^*_{bB}}{(C^*_{aB})^2} \end{pmatrix}.$$

We have negative definiteness of $D_A(s^*)$ on $H_A(s^*)$ if

$$\left(\frac{\partial f_A}{\partial M_A}(s^*) - \frac{\partial g_{aB}}{\partial C_{aB}}(s^*)\right) z_1 + \left(\frac{\partial g_{aA}}{\partial C_{aA}}(s^*) - \frac{\partial g_{aB}}{\partial C_{aB}}(s^*)\right) z_2 < 0 \text{ for all } z_1, z_2 > 0,$$

which is obviously satisfied. Hence, s^* is evolutionary stable.

Combining lemma 4.3.7 and 4.3.8, we conclude that s^* is globally stable.

□

We obtain the following sets of asymptotically stable states Δ^*:

$$\Delta^* = \begin{cases} \{\hat{s}(A)\} & \text{if } (1+\tau_{aB})(\tau_{bA} - (1-\phi_A)^2) < (1-\phi_B)^2 \\ \{\hat{s}(B)\} & \text{if } (1+\tau_{bA})(\tau_{aB} - (1-\phi_B)^2) < (1-\phi_A)^2 \\ \{s^*\} & \text{if } \begin{array}{l} (1+\tau_{aB})(\tau_{bA} - (1-\phi_A)^2) > (1-\phi_B)^2 \text{ and} \\ (1+\tau_{bA})(\tau_{aB} - (1-\phi_B)^2) > (1-\phi_A)^2. \end{array} \end{cases}$$

Notice that asymptotic stability coincides with strongness, as considered in Chapters 2 and 3. To show that the stationary state $\hat{s}(A)$ is strong whenever it is asymptotically stable, we follow the reasoning as in the proof of Proposition 2.4.2. Consider a coalition η of type b agents activating the market in region B as middlemen, and coalitions θ_a and θ_b of traders of type a and b, respectively, at market B. Let M be the size of coalition η and let C_a and C_b be the sizes of coalitions θ_a and θ_b, respectively.

A strict payoff improvement for all deviating traders from θ_a and θ_b is achieved if C_a, C_b exist such that

$$\begin{cases} (1-\phi_B)\frac{C_b}{C_a} - \tau_{aB} > \frac{1-\phi_A}{\hat{C}_{aA}} \\ \\ (1-\phi_B)\frac{C_a}{C_b} > (1-\phi_A)\hat{C}_{aA} - \tau_{bA}. \end{cases}$$

These inequalities fail to hold, in which case $\hat{s}$ is strong, if

$$\frac{1-\phi_B}{(1-\phi_A)^2} - \tau_{bA} \leq \frac{(1-\phi_A)^2 + \tau_{aB}(1-\phi_A)}{(1-\phi_B)(1-\phi_A)},$$

which yields the same condition for strongness as for asymptotic stability.

To conclude this section we consider the actual evolution of fractions over time. A typical picture is provided in Figure 4.1, where the middleman fractions are depicted. Three stages of evolution may be distinguished.

Stage 1

The region with the high access fee, say B, attracts middlemen more than the other region. As traders have not selected one of the markets explicitly, the payoff of a middleman is determined mainly by the access fee. Hence, the fraction M_B increases, whereas M_A decreases.

Stage 2

Traders become attracted to the region with the low access fee stronger: C_{aA} and C_{bA} increase, whereas C_{aB} and C_{bB} decrease.

Stage 3

The region with the high access fee becomes less attractive for middlemen, since traders leave this region in favor of the region with the low access fee. Middlemen go there also, until the state $\hat{s}(A)$ is reached.

This evolution of fractions illustrates the principle of 'survival of the fittest'. In the early stages of evolution, it is favorable for middlemen at the high access fee region to become active, since traders did not clearly select one of the regions to trade at. In later stages, the income source of middlemen at the high access fee region is 'destroyed', since traders become in favor of the other region. Then, it becomes 'fit' behavior for middlemen to follow the traders to that region.

In the context of a Prisoners' Dilemma, a more or less similar evolutionary pattern is described by Axelrod (1984), and Young and Foster (1991). They describe a population with agents adopting strategies for a repeated Prisoners' Dilemma in an evolutionary way. Young and Foster find that in early stages, coaltions of 'aggressive' players that are always defecting gain in size rapidly at the cost of 'nice' players that are always cooperating. In later stages, when 'aggressive' players almost only play against each other by the destruction of their source of high payoffs and therefore have low payoffs, players that cooperate in principle, but react to defection by also defecting, start to dominate the population. These players inhibit 'fit' behavior.

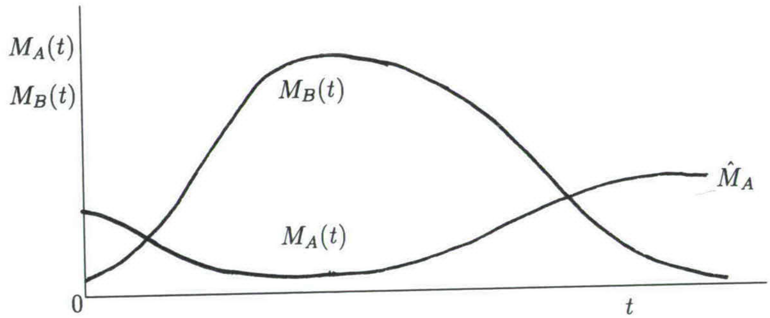

Figure 4.1: Evolution of middleman fractions

4.4 Market selection by traders

In order to obtain more insight in the process of selection of markets by traders, we analyze the evolution of trader fractions in a model without middlemen. The market formation process is thus ignored here. Also, transportation costs are assumed to be zero. We then have $C_{aA} + C_{aB} = 1$ and $C_{bA} + C_{bB} = 1$, so that a state is a vector (C_{aA}, C_{bA}) in the unit square $[0,1]^2$. The system of differential equations becomes

$$\begin{cases} \dot{C}_{aA} = (1-\phi_A)C_{bA} - (1-\phi_B)C_{aA} + (\phi_A - \phi_B)C_{aA}C_{bA} \\ \\ \dot{C}_{bA} = (1-\phi_A)C_{aA} - (1-\phi_B)C_{bA} + (\phi_A - \phi_B)C_{aA}C_{bA} \end{cases}$$

Suppose $\phi_A < \phi_B$. Then, the stationary states of the system are $(1,1)$ and $(0,0)$. We show that $(1,1)$ is the *globally* asymptotically stable state, that is, from every initial state $(C_{aA}(0), C_{bA}(0)) \in (0,1)^2$ the system converges to $(1,1)$ as time goes to infinity.

Therefore, consider the function $V : (0,1)^2 \to \mathbb{R}$ defined as $V(C_{aA}, C_{bA}) = C_{aA} + C_{bA}$. Obviously, V is maximized uniquely for $C_{aA} = C_{bA} = 1$. The time derivative function $\dot{V}$ of V is

$$\dot{V} = \dot{C}_{aA} + \dot{C}_{bA} = (\phi_B - \phi_A)(C_{aA} + C_{bA}) - 2(\phi_B - \phi_A)C_{aA}C_{bA}.$$

$\dot{V}$ is strictly positive on $(0,1)^2$. Therefore, V is a Lyapunov function, and $(1,1)$ is globally stable.

First, we consider the evolution of states to the asymptotically stable state $(1,1)$, characterized by all traders going to the market in region A. As an example, take $\tau_{aB} = \tau_{bA} = 0$, and $\phi_A = 0$. Then, $\hat{s}(A)$ is asymptotically stable whenever $\phi_B > 0$.

The curves γ_a and γ_b for which $\dot{C}_{aA} = 0$ and $\dot{C}_{bA} =$, respectively, are

$$\gamma_a = \left\{ (C_{aA}, C_{bA}) \in [0,1]^2 \mid C_{bA} = \frac{(1-\phi_B)C_{aA}}{1 - \phi_A + (\phi_A - \phi_B)C_{aA}} \right\}, \text{ and}$$

$$\gamma_b = \left\{ (C_{aA}, C_{bA}) \in [0,1]^2 \mid C_{bA} = \frac{(1-\phi-A)C_{aA}}{1 - \phi_B - (\phi_A - \phi_B)C_{aA}}. \right\}$$

The curves divide the state space into the areas I, II and III. In area I, we have $\dot{C}_{aA} > 0, \dot{C}_{bA} < 0$, in area II we have $\dot{C}_{aA}, \dot{C}_{bA} > 0$, and in area III we have $\dot{C}_{aA} < 0$ and $\dot{C}_{bA} > 0$. In Figure 4.2, the areas are illustrated for $\phi_A = 0$ and $\phi_B = 1/2$.

Any trajectory starting in area I or III enters the lense-shaped area II in finite time, after which it converges to the asymptotically stable state (1,1). Suppose we start in area III. In area III, a relatively small fraction of type b traders goes to market B. This means that these traders have relatively large gains-from-trade at market B. Although the transaction cost at market B is higher than at market A, the fraction C_{bB} is increasing, and hence C_{bA} decreasing, so that the trajectory moves towards area II. By the increase of the fraction C_{bB}, the gains-from-trade of type b consumers at market B decrease, however. If the trajectory reaches area II, the higher transaction cost of market B exactly outweighs the higher gains-from-trade. In area II, the transaction cost weighs heavier, so that the trajectory moves towards (1,1) from then on.

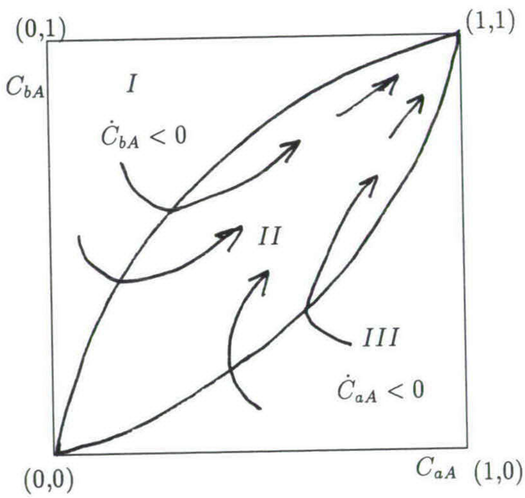

Figure 4.2: Trajectories

4.5 Proofs

Proof of Theorem 4.2.3

In Theorem 2.3.6, generic existence of a market equilibrium is shown where a subset of markets with sufficiently small access fees and set-up costs is active.

Lemma 4.5.1 *(cf. Theorem 2.3.6) Let be given numbers $\delta > 0$ and $\epsilon > 0$, and a subset $\Gamma^* \subset \Gamma$ with $\#\Gamma^* \leq n/2$ such that for every $\gamma \in \Gamma^*$ it market characteirstic $\psi(\gamma) = (\sigma, \phi)$ has the property that*

1. $||\phi^i|| < \delta$ *for all* $i \in N$.
2. $||\sigma^i|| < \epsilon$ *for all* $i \in N$.

If δ and ϵ are sufficiently small, then there generically exists a market equilibrium $\hat{s} \in \Delta$ such that all markets $\gamma \in \Gamma^*$ are active.

The equilibrium $\hat{s}$ has the following form.

1. $0 < \hat{s}^1_{\gamma_j m} < 1$ for every $\gamma_j \in \Gamma^* = \{\gamma_1, \ldots, \gamma_t\}$.
2. For every $1 \leq j \leq t$: $\hat{s}^i_{\gamma_j c} = 1$ for some $i \neq 1 \in I_j \subset N$, with $\#I_j \geq 2$.

We show that $\hat{s}$ is evolutionary stable. Therefore, we derive that the matrix of partial derivatives

$$\left(\frac{\partial f^1_{\gamma r}}{\partial s^1_{\gamma' r'}}(\hat{s}) \right)_{r,r' \in \{m,c\}, \gamma, \gamma' \in \Gamma}$$

is negative definite on the tangent space

$$H(\hat{s}) = \left\{ z \in \mathbb{R}^{2\#\Gamma} \left| \begin{array}{l} \sum_{\gamma \in \Gamma, r \in \{m,c\}} z_{\gamma r} = 0 \text{ and} \\ \hat{s}^1_{\gamma r} = 0 \text{ implies } z_{\gamma r} = 0, \gamma \in \Gamma, r \in \{m,c\} \end{array} \right. \right\}$$

at $\hat{s}$. The submatrix $A(\hat{s})$ which is derived from deleting the rows and columns of the matrix of partial derivatives corresponding to zero fractions of $\hat{s}$ is equal to

$$A(\hat{s}) = \begin{pmatrix} \frac{\partial f^1_{\gamma_1,m}}{\partial s^1_{\gamma_1,m}}(\hat{s}) & 0 & \cdots & 0 \\ 0 & \frac{\partial f^1_{\gamma_2,m}}{\partial s^2_{\gamma_2,m}}(\hat{s}) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{f^1_{\gamma_k,m}}{\partial s^k_{\gamma_t,m}}(\hat{s}) \end{pmatrix}.$$

Defining for $j \in \{1, \dots, t\}$ and $k \in L$: $M_j := s^1_{\gamma_j,m}$ and $y_{jk}(M_j) := \frac{\phi^1_{jk} I_j}{M_j}$, we have a payoff

$$f^1_{\gamma_j,m}(s) = u^1(w^1_1 - \sigma^1_{j1} + y_{j1}(M_j), \dots, w^1_l - \sigma^1_{jl} + y_{jl}(M_j)) \text{ for every } j \in \{1, \dots, t\},$$

so that

$$\frac{\partial f^1_{\gamma_j,m}}{\partial M_j}(\hat{s}) = -\sum_{k \in L} \frac{\partial u^i(w^1 - \sigma^1_j + y_j(\hat{M}_j))}{\partial y_{jk}(\hat{M}_j)} \times \frac{\phi^1_{jk} I_j}{(\hat{M}_j)^2} < 0, \tag{4.2}$$

by strict monotonicity of utility functions.

The matrix $A(\hat{s})$ is negative definite on $H(\hat{s})$ if

$$\left(\sum_{j=1}^{t-1} \frac{\partial f^1_{\gamma_j,m}}{\partial M_j}(\hat{s}) - \frac{\partial f^1_{\gamma_t,m}}{\partial M_t}(\hat{s}) \right) \sum_{j=1}^{t-1} M_j < 0,$$

which holds by (4.2). Therefore, $\hat{s}$ is evolutionary stable.

□

Proof of Theorem 4.2.5

In Theorem 2.3.9, existence of a market equilibrium is shown where a subset of markets with sufficiently small access fees is active, if the endowment of one type is sufficiently small.

Lemma 4.5.2 *(cf. Theorem 2.3.9)*

Let be given numbers $\delta > 0$ and $\epsilon > 0$, a type $k \in N$, and a non-empty subset $\Gamma^* \subset \Gamma$ with $\#\Gamma^* \leq n/2$ such that for all $\gamma \in \Gamma^*$ their market characteristic $\psi(\gamma) = (\sigma, \phi)$ and endowments w^i, $i \in N \setminus \{k\}$ have the property that

1. $||\phi^i|| < \delta$ *for all* $i \in N$.
2. $\frac{||w^k||}{||w^i||} < \epsilon$.

If δ and ϵ are sufficiently small, then there generically exists a market equilibrium $\hat{s} \in \Delta$ with the property that all markets $\gamma \in \Gamma^*$ are active.

The market equilibrium $\hat{s} \in \widetilde{\Delta}$ has the following form.

1. $0 < \hat{s}^1_{\gamma_j m} < 1$ for every $\gamma_j \in \Gamma^* = \{\gamma_1, \dots, \gamma_t\}$.
2. $\hat{s}^1_{\gamma_1 c} = 1 - \sum_{j=1}^t \hat{s}^1_{\gamma_1 m}$.
3. For every $1 \leq j \leq t$: $\hat{s}^i_{\gamma_j c} = 1$ for some $i \in I_j \subset N$, with $\# I_j \geq 2$.

For expositional ease, we show the result for access fees $\phi^i_j = (\phi_j, 0, \dots, 0) \in \mathbb{R}^l_+$, for $i \in N, j \in \{1, \dots, t\}$. The result generalizes by continuity. Define $\phi := (\phi_1, \phi_2, \dots, \phi_t)$.

We introduce

$$M_j := s^1_{\gamma_j m}, C := s^1_{\gamma_1 c}, \text{ and } \alpha_j := \sum_{i \in I_j} \phi_i, 1 \leq j \leq t.$$

We show that the matrix of partial derivatives

$$\left(\partial f^1_{\gamma r} / \partial s^1_{\gamma' r'}(\hat{s})\right)_{r, r' \in \{m,c\}, \gamma, \gamma' \in \Gamma}$$

is negative definite on the tangent space $H(\hat{s})$ at $\hat{s}$. The submatrix which is derived from deleting the rows and columns of the matrix of partial derivatives corresponding to zero fractions is

$$\begin{pmatrix} \frac{\partial f^1_{\gamma_1,m}}{\partial M_1}(\hat{s}) & 0 & \cdots & 0 & \frac{\partial f^1_{\gamma_1,m}}{\partial C}(\hat{s}) \\ 0 & \frac{\partial f^1_{\gamma_2,m}}{\partial M_2}(\hat{s}) & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \frac{\partial f^1_{\gamma_t,m}}{\partial M_t}(\hat{s}) & 0 \\ 0 & 0 & \cdots & 0 & \frac{\partial f^1_{\gamma_t,c}}{\partial C}(\hat{s}) \end{pmatrix}.$$

We have negative definiteness on the tangent space at $\hat{s}$ if

$$\sum_{j=1}^{t}\left\{(M_j)^2\frac{\partial f^1_{\gamma_j,m}}{\partial M_j}(\hat{s})\right\} - M_1\sum_{j=1}^{t} M_j\frac{\partial f^1_{\gamma_j,m}}{\partial C}(\hat{s}) + \left(\sum_{j=1}^{t} M_j\right)^2 \frac{\partial f^1_{\gamma_1,c}}{\partial C}(\hat{s}) < 0 \tag{4.3}$$

for every $s \in H(\hat{s})$.

Therefore, we show that for every $j \in \{1,\ldots,t\}$

$$\lim_{\phi_j \downarrow 0} \frac{\partial f^1_{\gamma_j,m}}{\partial M_j}(\hat{s}(\phi)) = -\infty,$$

by means of the following lemmas.

Lemma 4.5.3 *For every $j \in \{1,\ldots,t\}$ we have $\lim_{\phi_j\downarrow 0} \frac{\phi_j}{\hat{M}_j(\phi)} > 0$.*

PROOF

Suppose $\lim_{\phi_j\downarrow 0} \frac{\phi_j}{\hat{M}_j(\phi)} = 0$, then

$$\lim_{\phi_j \downarrow 0} f^i_{\gamma_j,m}(\hat{M}_j(\phi)) = u^i(w^i - \sigma^i_j) < u^i(w^i),$$

which contradicts the existence of a market equilibrium for sufficiently small access fees, i.e.,

$$\lim_{\phi_j \downarrow 0} f^i_{\gamma_j,c}(\hat{s}(\phi)) \geq u^i(w^i).$$

□

A direct consequence of Lemma 4.5.3 is the following lemma.

Lemma 4.5.4 *For every $j \in \{1,\ldots,t\}$ we have $\lim_{\phi_j\downarrow 0} \hat{M}_j(\phi) = 0$.*

For $s \in \widetilde{\Delta}$ and $j \in \{1,\ldots,t\}$ define the vector

$$z_j(s) = (w_1^1 - \sigma_{j1}^1 + y_j(s), w_2^1 - \sigma_{j2}^1, \dots, w_l^1 - \sigma_{jl}^1),$$

with

$$y_j(s) := \begin{cases} \frac{\alpha_1 + \phi_1 C}{M_1} & \text{if } j = 1 \\ \frac{\alpha_j}{M_j} & \text{if } j \in \{2, \dots, t\}. \end{cases}$$

We have $f^1_{\gamma_j,m}(s) = u^1(z_j(s))$ for $j \in \{1, \dots, t\}$. The partial derivative with respect to M_j for $j \in \{1, \dots, t\}$ in $\hat{s}$ are therefore

$$\frac{\partial f^1_{\gamma_j,m}(\hat{s})}{\partial M_j} = \begin{cases} -\frac{\partial u^1(z_j(\hat{s}))}{\partial y_j(\hat{s})} \times \frac{\alpha_1 + \phi_1 \hat{C}}{(\hat{M}_1)^2} & \text{if } j = 1 \\ \\ -\frac{\partial u^1(y_j(\hat{s})}{\partial y_j(\hat{s})} \times \frac{\alpha_j}{(\hat{M}_j)^2} & \text{otherwise.} \end{cases}$$

For $j = 1$, Lemmas 4.5.3 and 4.5.4 imply

$$\lim_{\phi_1 \downarrow 0} \frac{\partial f^1_{\gamma_1,m}}{\partial M_1}(\hat{s}(\phi)) = \lim_{\phi_1 \downarrow 0} -\frac{\alpha_1}{(\hat{M}_1(\phi))^2} + \frac{\phi_1}{\hat{M}_1(\phi)} \times \frac{\hat{C}(\phi)}{\hat{M}_1(\phi)} = -\infty.$$

Analogously we find

$$\lim_{\phi_j \downarrow 0} \frac{\partial f^j_{\gamma_j,m}(\hat{s}(\phi))}{\partial M_j} = -\infty \text{ for } j \in \{2, \dots, t\}.$$

Moreover,

$$\frac{\partial f^1_{\gamma_j,m}(\hat{s})}{\partial C} = \frac{\partial u^1(z_1)}{\partial y_1}(\hat{s}) \times \frac{\phi_1}{\hat{M}_1(\phi)} > 0,$$

and

$$\frac{\partial f^1_{\gamma_1,c}(\hat{s})}{\partial C} < \infty$$

by differentiability of the function $f^1_{\gamma_1 c}$.

A sufficient condition for (4.3) to be satisfied is

$$\eta(\hat{s})\sum_{j=1}^{t}(M_j)^2 > \theta(\sum_{j=1}^{t} M_j)^2,$$

where

$$\eta(\hat{s}) := \left\{-\frac{\partial f^1_{\gamma_{j^*},m}(\hat{s})}{\partial M_{j^*}}\right\} = \min_{j\in\{1,\dots,t\}}\left\{-\frac{\partial f^1_{\gamma_j,m}(\hat{s})}{\partial M_j}\right\}, \text{ and}$$

$$\theta := \max_{s\in\widetilde{\Delta}}\{-\frac{\partial f^1_{\gamma_j,c}(s)}{\partial C}\} < \infty.$$

If $\eta > 2\theta$, we have

$$\eta(\hat{s})\textstyle\sum_{j=1}^{t}(M_j)^2 > \theta(\sum_{j=1}^{t} M_j)^2 \text{ if}$$

$$(\eta(\hat{s})-\theta)\textstyle\sum_{j=1}^{t}(M_j)^2 - 2\theta\sum_{i=1}^{t}\sum_{j=i+1}^{t} M_iM_j > (\eta(\hat{s})-\theta)(\sum_{j=1}^{t}(M_j))^2 > 0.$$

Since $\lim_{\phi_{j^*}\downarrow 0}\eta(\hat{s}) = \infty$, and $\theta < \infty$, this implies that (4.3) is satisfied for sufficiently small ϕ_{j^*}.

Proof of Proposition 4.3.4

To derive the stability condition for $\hat{s}(A)$, we determine the matrix of partial derivatives

$$J(\hat{s}(A)) = \left(\frac{\partial \dot{C}_{ij}}{\partial C_{kl}}(\hat{s}(A))\right)_{i,j\in\{a,b\},\ k,l\in\{A,B\}}$$

in equilibrium. In order to determine its eigenvalues, we derive the determinant of the matrix

$$\begin{pmatrix} -1-\lambda & \tau_{aB}(1-\phi_A) & 0 & (1-\phi_A)(1-\phi_B) \\ 0 & -(\tau_{aB}+1)-\lambda & 0 & (1-\phi_B) \\ 0 & -1 & \tau_{bA}-(1-\phi_A)^2-\lambda & 0 \\ 0 & (1-\phi_B) & 0 & \tau_{bA}-(1-\phi_A)^2-\lambda \end{pmatrix}.$$

The eigenvalues are derived from the characteristic equation

$$(-1-\lambda)(\tau_{bA}-(1-\phi_A)^2-\lambda)(\lambda^2-(\alpha+\gamma)\lambda-(\beta^2-\alpha\gamma)=0,$$

with $\alpha=-(\tau_{aB}+1)$, $\beta=1-\phi_B$ and $\gamma=\tau_{bA}-(1-\phi_A)^2$. The characteristic equation yields us eigenvalues

$$\lambda_1=-1,$$

$$\lambda_2=\tau_{bA}-(1-\phi_A)^2,$$

$$\lambda_3=(\alpha+\gamma)/2+\sqrt{(\alpha+\gamma)^2-4(\alpha\gamma-\beta^2)}/2, \text{ and}$$

$$\lambda_4=(\alpha+\gamma)/2-\sqrt{(\alpha+\gamma)^2-4(\alpha\gamma-\beta^2)}/2$$

We have stability if all eigenvalues are strictly negative. This is the case if $\lambda_2<0$ and $\lambda_3<0$. That is, if

$$\begin{cases} \tau_{aB}+\tau_{bA}<(1-\phi_A)^2+1 \\ \tau_{bA}<(1-\phi_A)^2 \\ (1-\tau_{aB})((1-\phi_A)^2-\tau_{bA})>(1-\phi_B)^2, \end{cases}$$

which is satisfied if $(1-\tau_{aB})((1-\phi_A)^2-\tau_{bA})>(1-\phi_B)^2$.

$\square$

Chapter 5

Intermediated bilateral matching

5.1 Introduction

In this chapter, which is based on van Raalte and Webers (1995), we study bilateral matching with costly intermediation. With bilateral matching, there are two types of agents, each agent seeking to be matched with an agent of the other type, to exchange commodities or services. The real-estate market is an example. There, buyers typically seek one seller of a house. Another example is the marriage market; men desire to be matched with women one-to-one and vice versa. As a third example, we mention companies searching for entertainers to perform at special occasions.

It may be difficult or impossible for market participants to establish matches on their own account. Problems are incurred mainly in obtaining information about desirable matching partners. Special skills may be required for extracting the right information, as found with intermediating agencies, such as real estate brokers, dating agencies and theater agencies. Brokers are examples of *match makers*, the intermediating institutions in this chapter. Match makers bring agents of both market sides together, without being involved in the interaction between them. In the housing market, for instance, brokers rather than buying houses and reselling them again, provide sellers the opportunity to sell houses themselves. Match makers differ from *market makers* in this respect. E.g., Rubinstein and Wolinsky (1987), and Biglaiser and Friedman (1993) analyze the behavior of market makers. Yavas (1993) compares the effectiveness of match making and market making for different types of markets.

We study a model with two market parties (buyers and sellers), each distributed uniformly on a circle, both having possibly different densities. Each buyer desires to be matched with a seller. Unable to establish these matches on their own account, buyers and sellers are matched by one of two middlemen (match makers), located symmetrically on the circle, that is, diametrically opposite to each other.

Every agent going to a middleman pays him a commission fee, and incurs a certain relational cost[1], provided he is matched. Relational costs are proportional to the distance along the circle to a middleman[2]. Furthermore, each agent has a reservation price, indicating how much he is willing to spend, in terms of fee plus relational cost, in order to be matched. The reservation price may differ over types.

With respect to buyers and sellers, we make the assumption that each of them goes to the middleman whose sum of fee and relational cost is the smaller. Also, the middlemen expect the agents to behave that way. We do not incorporate more sophisticated expectations of middlemen with respect to agents' behavior, or of the agents with respect to the other agents. If they would do so, the risk of not being matched would enter the agents' utility function[3]. We think there is no strong reason to incorporate such expectations, since neither the middlemen nor the agents possess any *a priori* information, based on which they could form any reasonable expectations about the other agents' behavior. As in the previous chapters, we encounter a considerable inertia associated with markets. Namely, the inability of individual agents to obtain gains-from-trade. Because we focus on the competition in commission fees by middlemen, we choose not to complicate the model unnecessarily. It should be noted, that the behavioral assumptions made are consistent with strategic behavior. Given that the middlemen set equilibrium fees, the buyers and sellers cannot do better than indeed go to the 'cheapest' middleman.

[1] Gilles, Diamantaras and Ruys (1996) incorporate relational costs in a model of costly trade infrastructures carrying a Walrasian market. In that model, relational costs are the costs for consumers using the infrastructure.

[2] Therefore, the model can be seen as a variant of the Salop (1979) model of spatial competition, where transportation costs take the place of relational costs.

[3] In some sense, this risk could be related to the risk associated with the timely delivery of products (see Espinosa (1992)).

The game of setting commission fees played by the middlemen is analyzed for two different cases. First, the case where densities of sellers and buyers are equal. Second, the case where densities are unequal, specifically, if the density of sellers is smaller than the density of buyers. The two cases yield structurally different sets of equilibria when both reservation prices are sufficiently large, in which case there is competition for those agents that are indifferent between going to either one of the middlemen. If reservation prices are sufficiently low, there is no such competition, so that the middlemen establish 'local monopolies' in equilibrium. Then, obviously, the analysis is identical for both cases. We show that, generically, there exists a unique Nash equilibrium for the game when densities are unequal, whereas in case of equal densities, multiple equilibria may exist when competition occurs. In equilibrium, fees are such that every middleman has an equal share of buyers and sellers[4]. This turns out to be the reason for the indeterminacy in the equal density case. Then, namely, the middlemen compete for both types of agents. If densities are unequal, only competition for one type occurs, namely the type with the lowest density, e.g., the sellers. Sellers then entirely determine the fee for buyers; the indeterminacy is resolved.

Among the equilibria, we find equilibria in which one type is charged a zero fee. The match makers even desire to subsidize this type, that is, charge a negative fee. In the local monopoly case, this kind of equilibrium occurs if the difference in reservation prices is sufficiently large. In that case, the type with the lower reservation price is 'free-riding'. The free-riding phenomenon arises from the fact that the payoff of a middleman is determined by the minimum of his shares of buyers and sellers. To increase his payoff, he needs to increase his shares of both types. In order to achieve this, the type with the low reservation price should be charged a relatively low fee, or even a nonpositive fee. In that case, the 'low' type is needed only to attract the 'high' type, from which positive fees are collected.

The free-riding phenomenon is also found outside the local monopoly region. For the case with unequal densities, equilibria may exist where the short side of the market is served for free, under competition. The reason is that the middlemen compete for an indifferent agent only on the short side. The fee for the agents on the long side is chosen to adjust

[4]Shares could be interpreted as demand and supply in certain markets. It is often assumed in the literature, that either supply is not binding or the demand functions are exogenous. In our model, demand functions are endogenous. The model can be seen as a 'strategic market coverage' type. Strategic market coverage through advertising was considered by Boyer and Moreaux (1993).

the share of these agents to the share of short side agents. A real-life example of a market with an advantageous position of the agents on the short side is the housing market. In the U.S.A., e.g., often the real-estate brokers only charge a fee to the sellers when they form the long side of the market. Another example are dating agencies, where, usually, females are attracted by 'subsidizing' them.

The remainder of the chapter is organized as follows. In Section 2, the model is formulated, and the different types of competition are introduced. Section 3 discusses the case of equal densities for both types. Section 4 considers the case of unequal densities. Proofs are gathered in Section 5.

5.2 The model

There is a continuum of buyers and sellers distributed uniformly along a circle with perimeter one. Sellers are distributed with density $\alpha_S > 0$ along the circle, and buyers with density $\alpha_B > 0$. Without loss of generality, we assume $\alpha_S \leq \alpha_B$. In the sequel, we sometimes call sellers type 1 agents, and buyers type 2 agents. Sellers and buyers desire to be matched. In order to be matched, they need an intermediating institution; it is too difficult for them to establish matches on their own account, for instance because obtaining information about desirable matching partners is too costly. There are two of such intermediaries, further referred to as *middlemen*, indexed $i \in I = \{1, 2\}$. Middleman $i \in I$ is located along the circle at $x_i = \frac{i-1}{2}$. This means, middleman are located diametrically on the circle. Middleman i charges a *commission fee* $\phi^i_j > 0$ for providing a matching service to a type $j \in J = \{B, S\}$ agent.

There are *relational costs* between buyers and sellers on the one hand and middlemen on the other. Namely, buyers and sellers face identical linear relational costs with unit cost $t > 0$; an agent at distance x from a middleman incurs a cost $t \cdot x$ when making use of that middleman's service.

Furthermore, every agent of type $j \in J$ has a reservation price $\overline{p}_j > 0$ for the relational cost and fee charged by any of the two middlemen, i.e., they want to pay up to an amount $\overline{p}_j$ in order to be matched. Reservation prices are assumed to be given exogenously. Let $\phi^i \in \mathcal{P} = [0, \overline{p}_S] \times [0, \overline{p}_B]$ denote the tuple of fees (ϕ^i_S, ϕ^i_B) of middleman $i \in I$. It may happen that the sum of fee and relational costs are so high that the reservation price of some type cannot cover these. The set of covered agents for a certain fee is called potential market area.

Definition 5.2.1 *For middleman $i \in I$, the* **potential market area** *of agents of type $j \in J$ at fee ϕ_j^i, denoted by $M_j^i(\phi_j^i)$, is the set of agents of type j for which the sum of relational cost and fee ϕ_j^i charged by middleman i does not exceed the reservation price $\overline{p}_j$.*

More formally, $M_j^1(\phi_j^1) = \{a \in [0,1] \mid \phi_j^1 + t \cdot a \leq \overline{p}_j \text{ or } \phi_j^1 + t \cdot (1-a) \leq \overline{p}_j\}$ and $M_j^2(\phi_j^2) = \{a \in [0,1] \mid \phi_j^2 + t \cdot (\frac{1}{2} - a) \leq \overline{p}_j \text{ or } \phi_j^2 + t \cdot (a - \frac{1}{2}) \leq \overline{p}_j\}$ for $j \in J$. Notice that for each middleman both potential market areas form an interval. The notion of potential market areas is used to describe the structure of competition among the two middlemen. We distinguish between three different situations.

Definition 5.2.2 *At given fees, there is* **strong competition** *if the potential market areas for the two middlemen have a nonempty intersection for both types of agents. There is* **weak competition** *if the potential market areas for the middlemen have a nonempty intersection for one of the two types of agents and for the other type the intersection is either a point or empty. There is* **no competition** *if the potential market areas of the middlemen have an intersection which is either a point or empty for both types of agents.*

For middleman $i \in I$, the size of the potential market area of agents of type $j \in J$ at fee ϕ_j^i is the length of the interval of agents of type j for which the sum of the relational cost to middleman i and the fee of middleman i, ϕ_j^i, does not exceed the reservation price $\overline{p}_j$. For middleman $i \in I$, at fees ϕ^1 and ϕ^2, the minimum of the sizes of the potential market area and the competitive market area of agents of type $j \in J$ is denoted by $X_j^i(\phi^1, \phi^2)$. The *market size* $X^i(\phi^1, \phi^2)$ for middleman $i \in I$, at fees ϕ^1 and ϕ^2, is equal to $min_{j \in J}\ X_j^i(\phi^1, \phi^2)$.

It is easy to verify that for both middlemen the potential market areas of agents of type j have a nonempty intersection in case $\frac{\phi_j^1 + \phi_j^2}{2} + \frac{t}{4} \leq \overline{p}_j$ and have an intersection which is either a point or empty in case $\frac{\phi_j^1 + \phi_j^2}{2} + \frac{t}{4} \geq \overline{p}_j$. This means that there are four different regions under concern.

For $\frac{\phi_S^1+\phi_S^2}{2}+\frac{t}{4}\geq\overline{p}_S$ and $\frac{\phi_B^1+\phi_B^2}{2}+\frac{t}{4}\geq\overline{p}_B$ we have the situation of no competition. The market size for middleman $i\in I$ is given then by

$$X^i(\phi^i,\phi^k)=\min\left\{\alpha_S\left(\frac{2(\overline{p}_S-\phi_S^i)}{t}\right),\alpha_B\left(\frac{2(\overline{p}_B-\phi_B^i)}{t}\right)\right\}. \tag{5.1}$$

For $\frac{\phi_S^1+\phi_S^2}{2}+\frac{t}{4}\leq\overline{p}_S$ and $\frac{\phi_B^1+\phi_B^2}{2}+\frac{t}{4}\geq\overline{p}_B$ we have the situation of weak competition, where the middlemen compete for sellers. The market size for middleman $i\neq k\in I$ is given then by

$$X^i(\phi^i,\phi^k)=min\left\{\alpha_S\left(\frac{1}{2}+\frac{\phi_S^k-\phi_S^i}{t}\right),\alpha_B\left(\frac{2(\overline{p}_B-\phi_B^i)}{t}\right)\right\}. \tag{5.2}$$

For $\frac{\phi_S^1+\phi_S^2}{2}+\frac{t}{4}\geq\overline{p}_S$ and $\frac{\phi_B^1+\phi_B^2}{2}+\frac{t}{4}\leq\overline{p}_B$ we have the situation of weak competition, where the middlemen compete for buyers. The market size for middleman $i\neq k\in I$ is given then by

$$X^i(\phi^i,\phi^k)=min\left\{\alpha_S\left(\frac{2(\overline{p}_S-\phi_S^i)}{t}\right),\alpha_B\left(\frac{1}{2}+\frac{\phi_B^k-\phi_B^i}{t}\right)\right\}. \tag{5.3}$$

Finally, for $\frac{\phi_S^1+\phi_S^2}{2}+\frac{t}{4}\leq\overline{p}_S$ and $\frac{\phi_B^1+\phi_B^2}{2}+\frac{t}{4}\leq\overline{p}_B$ we have the situation of strong competition. The market size for middleman $i\neq k\in I$ is given then by

$$X^i(\phi^i,\phi^k)=min\left\{\alpha_S\left(\frac{1}{2}+\frac{\phi_S^k-\phi_S^i}{t}\right),\alpha_B\left(\frac{1}{2}+\frac{\phi_B^k-\phi_B^i}{t}\right)\right\}. \tag{5.4}$$

Given that only buyers and sellers actually matched pay a fee, given fees ϕ^1,ϕ^2 middleman i's payoff is equal to

$$\Pi^i(\phi^i,\phi^k)=(\phi_S^i+\phi_B^i)\ X^i(\phi^i,\phi^k). \tag{5.5}$$

We look for a Nash equilibrium of the game in which the middlemen simultaneously choose fees as to maximize their payoffs. The game is solved by fees $\hat{\phi}^i\in\mathcal{P}$, $\hat{\phi}^k\in\mathcal{P}$ satisfying

$$\Pi^i(\hat{\phi}^i,\hat{\phi}^k)\geq\Pi^i(\phi^i,\hat{\phi}^k)$$

for all $\phi^i \in \mathcal{P}$ and $k \neq i \in I$.

In the sequel, the game in which middlemen simultaneously choose fees is referred to as G. In the game G, middleman $i \in I$ chooses fees ϕ^i_j, $j \in J$, as to maximize its payoff given the choice of middleman $k \neq i$. Because middlemen are located symmetrically it makes sense to look for an equilibrium in which both middlemen choose the same fees. Moreover, for each middleman, the shares of buyers and sellers must be equal in equilibrium. This is stated in the following lemma.

Lemma 5.2.3 *In any Nash equilibrium $(\hat{\phi}^i, \hat{\phi}^k) \in \mathcal{P} \times \mathcal{P}$ of the game G, for middleman $i \in I$ his shares of buyers and sellers are equal.*

PROOF

Suppose some middleman's share of type $j \in J$ agents is higher than its share of the other type of agents. Then, increasing the fee for the type j agents increases payoffs since the share of the other type of agents does not change.

$\square$

5.3 The case of equal densities

In this section, we analyze the equilibria in case of equal densities for buyers and sellers. We first state a general existence result. After that, we specify the equilibria in more detail.

Theorem 5.3.1 *For the case $\alpha_S = \alpha_B$ there exists a unique Nash equilibrium for the game G if reservation prices are either sufficiently different, or sufficiently small. Otherwise, there either exists one continuum of equilibria or there exist two or three continua of equilibria.*

Essentially, we can distinguish two types of equilibria. First, equilibria in which both types are charged a strictly positive fee. Second, equilibria in which one type of agents is charged a zero fee, that is, served for free.

Proposition 5.3.2 *There are regions of reservation prices for which the corresponding equilibrium fee is zero for some type of agents.*

More specifically, four different regions can be distinguished in which one type of agents is served for free. In two of these regions, we have a regime of no competition. There, the type with the low reservation price is served for free. Middlemen actually desire to subsidize the type with the low reservation price. The type of agents with the high reservation price is able to pay a higher fee than the other type. Competition drives fees down to zero for the type with the low reservation price and only the type with the high reservation price brings in a positive payoff. The fact that payoffs are determined by the minimum of both shares, which can be seen as a market externality associated with bilateral matching, means that an equal share of the type of agents with the low reservation price has to be attracted. Because of their relatively low willingness to pay, the middlemen have to charge these agents a low fee in order to attract them. Nevertheless, the negative effect on payoffs of the zero fee is more than compensated by the positive effect on the share of types with a low reservation price.

In the other two regions in which one type is served for free, we have a regime of strong competition. One type is served for free and the other type is charged a relatively high fee between certain bounds. Due to the fact that the fees for one type are zero, no middleman gains by charging the other type a lower fee, because his market size is not increased. Due to symmetry, any of the two types of agents may be served for free.

In the following corollary, a limit result for zero relational costs is stated, in which case we have 'pure' Bertrand competition between middlemen.

Corollary 5.3.3 *For the limit case $t = 0$, some type is served for free in any equilibrium. Thereby, $\hat{\phi}^1 = \hat{\phi}^2 = \langle \phi, 0 \rangle$ with $\phi \in [0, \overline{p}_S]$, or $\hat{\phi}^1 = \hat{\phi}^2 = \langle 0, \phi' \rangle$ with $\phi' \in [0, \overline{p}_B]$.*

The intuition for this result is that the middleman with the lowest fees attracts all agents in case relational costs are absent, since then the fee is the only relevant variable for the agents. An undercutting argument yields that no equilibrium exists in which the fees for both types are strictly positive. If, however, only one fee is strictly positive, it is not possible to undercut both fees. Furthermore, charging higher fees is not optimal because of the strong effect of the market size on payoffs. Notice that although there is pure Bertrand competition, payoffs may be positive by the market externality.

For the special case where both types have equal reservation prices, we find that charging buyers and sellers the same fees is an equilibrium strategy.

Corollary 5.3.4 *Suppose $\overline{p}_S = \overline{p}_B = \overline{p}$. Then, for any $\overline{p} \in \mathbb{R}_+$ there exists a Nash equilibrium in which buyers and sellers are served at the same fee. This equilibrium is unique if $\overline{p}$ is sufficiently small.*

In Table 5.1, the different equilibria in case of equal densities are given. The areas I, II^a, II^b, III, and IV^a are illustrated graphically in Figure 5.1. The areas IV^b and IV^c are illustrated graphically in Figure 5.2.

Area	Fees	Profits
I	$\langle \frac{3\overline{p}_S-\overline{p}_B}{4}, \frac{3\overline{p}_B-\overline{p}_S}{4} \rangle$	$\frac{\alpha}{4t}(\overline{p}_S+\overline{p}_B)^2$
II^a	$\langle \overline{p}_S-\overline{p}_B, 0 \rangle$	$\frac{2\alpha}{t}\,\overline{p}_B(\overline{p}_S-\overline{p}_B)$
II^b	$\langle 0, \overline{p}_B-\overline{p}_S \rangle$	$\frac{2\alpha}{t}\,\overline{p}_S(\overline{p}_B-\overline{p}_S)$
III	$\langle \overline{p}_S-\frac{t}{4}, \overline{p}_B-\frac{t}{4} \rangle$	$\frac{\alpha}{2}(\overline{p}_S+\overline{p}_B-\frac{t}{2})$
IV^a	$\langle \varphi^a, t-\varphi^a \rangle$	$\frac{\alpha}{2}t$
IV^b	$\langle 0, \varphi^b \rangle$	$\frac{\alpha}{2}\varphi^b$
IV^c	$\langle \varphi^c, 0 \rangle$	$\frac{\alpha}{2}\varphi^c$

Table 5.1: Equilibrium fees and payoffs for the different regions in case $\alpha_S = \alpha_B = \alpha$, with $\varphi^a \in [0, \overline{p}_S - \frac{t}{4}] \cap [\frac{5t}{4} - \overline{p}_B, t]$, $\varphi^b \in [t, \overline{p}_B - \frac{t}{4}]$, and $\varphi^c \in [t, \overline{p}_S - \frac{t}{4}]$.

We can distinguish three areas of no competition, one area of weak competition, and three areas of strong competition. It is easily checked that the corresponding fees and payoffs change continuously in and between the areas.

Areas I, II^a, II^b: No competition.

In the areas I, II^a, and II^b, the reservation price of at least one of the types of agents is so low that both middlemen establish 'local monopolies'. In area I, the differences between the reservation prices of sellers and buyers are sufficiently low to obtain an equilibrium with both fees positive. In equilibrium, the fees are such that the agents with the higher reservation price also pay a higher fee. In areas II^a and II^b, the fee for one type is zero.

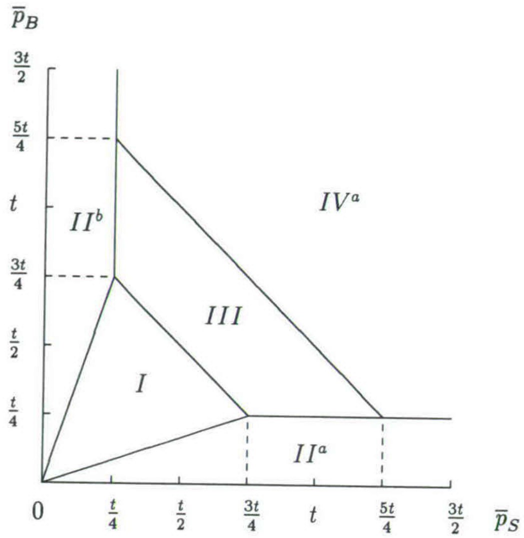

Figure 5.1: Regions I to IV^a in case $\alpha_S = \alpha_B$.

Area III: Weak competition.

In area III, the reservation prices are sufficiently high to create a situation of weak competition. In this area, the situations of strong competition, weak competition and no competition coincide.

Areas IV^a, IV^b, IV^c: Strong competition.

For the situation of strong competition, different types of continua of equilibria may coexist. For reservation prices in area IV^a, there is a continuum of equilibria where the fees, with sum equal to t, can be divided in an arbitrary way. In this equilibrium, in general, no agent is charged a very high price compared to its reservation price. Notice that also a 'fair' treatment of agents, that is, $\varphi = \frac{t}{2}$, is allowed as an equilibrium. The situation where one of the types of agents is charged a very high price compared to its reservation price occurs, however, in the two other continua of equilibria for the areas IV^b and IV^c. In these areas equilibria exist in which one type of agents is served for free and the other type is charged a very high price compared to its reservation price. Equilibria in area IV^a, in which case there is an upper bound on the payoffs, thus may

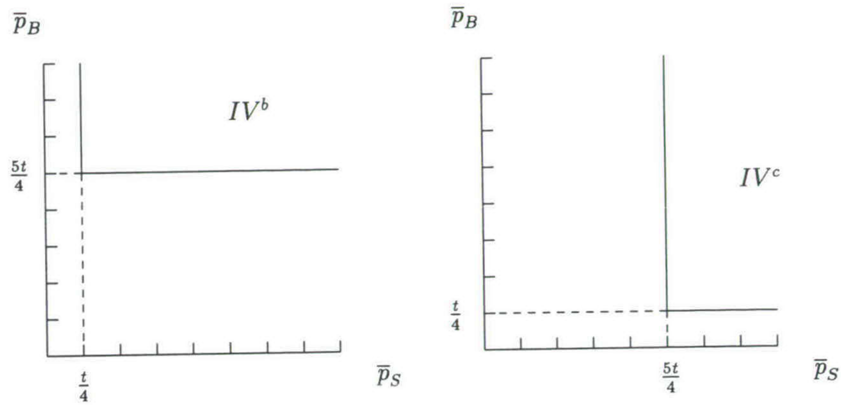

Figure 5.2: Regions IV^b and IV^c in case $\alpha_S = \alpha_B$

coexist with equilibria in area IV^b or area IV^c, in which case there exist equilibria for which the payoffs tend to infinity if the appropriate reservation price tends to infinity.

For the special case $\overline{p}_S = \overline{p}_B = \overline{p}$ equilibrium fees as a function of the reservation price $\overline{p}$ are drawn in Figure 5.3. We see that both types of agents are charged the same fee for relatively low values of $\overline{p}$, i.e., $0 \leq \overline{p} \leq \frac{3t}{4}$. For relatively high values of the reservation price $\overline{p}$, i.e., $\overline{p} > \frac{3t}{4}$, there is one continuum of equilibria for $\frac{3t}{4} < \overline{p} < \frac{5t}{4}$ and there are three continua of equilibria for $\overline{p} \geq \frac{5t}{4}$.

5.4 The case of unequal densities

In this section, we analyze the case $\alpha_S < \alpha_B$. The introduction of asymmetry causes the continua of equilibria found with equal densities to disappear.

Theorem 5.4.1 *For the case $\alpha_S < \alpha_B$, generically, there exists a unique Nash equilibrium for the game G.*

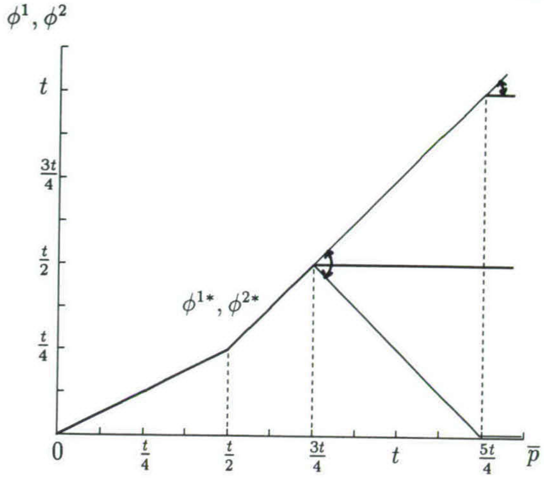

Figure 5.3: Equilibrium fees in case $\alpha_S = \alpha_B$ and $\overline{p}_S = \overline{p}_B = \overline{p}$.

Similar to the case with equal densities, we find equilibria in which one type of agents is served for free.

Proposition 5.4.2 *There are regions of reservation prices for which the corresponding equilibrium fee is zero for some type of agents.*

Also, in the limit case of pure Bertrand competition, at least one type is served for free.

Corollary 5.4.3 *In the limit case $t = 0$, at least one type is served for free in any Nash equilibrium. Thereby, $\hat{\phi}^1 = \hat{\phi}^2 = \langle \phi, 0 \rangle$ with $\phi \in [0, \overline{p}_S]$ if $\overline{p}_S \geq 0$ and $\overline{p}_B = 0$, and $\hat{\phi}^1 = \hat{\phi}^2 = \langle 0, \overline{p}_B \rangle$ if $\overline{p}_S \geq 0$ and $\overline{p}_B > 0$.*

For the special case where both types of agents have equal reservation prices, we find that the short side of the market has an advantage over the long side of the market in the sense that sellers pay a lower fee than buyers. This reflects the market externality inherent in bilateral matching.

Corollary 5.4.4 *Suppose $\overline{p}_S = \overline{p}_B = \overline{p}$. In any Nash equilibrium and for all $\overline{p} \in \mathbb{R}_+$, sellers are served at a lower fee than buyers. If $\overline{p}$ is relatively large, sellers are served for free.*

In Table 5.2, the different equilibria in case of unequal densities are given. The different areas are illustrated graphically in Figure 5.4.

Area	Fees	Profits
I	$\langle \frac{(2\alpha_S+\alpha_B)\overline{p}_S-\alpha_B\overline{p}_B}{2(\alpha_S+\alpha_B)}, \frac{(\alpha_S+2\alpha_B)\overline{p}_B-\alpha_S\overline{p}_S}{2(\alpha_S+\alpha_B)} \rangle$	$\frac{\alpha_S\alpha_B}{2(\alpha_S+\alpha_B)t}(\overline{p}_S+\overline{p}_B)^2$
II^a	$\langle \overline{p}_S - \frac{\alpha_B}{\alpha_S}\,\overline{p}_B, 0 \rangle$	$2\frac{\alpha_B}{t}\,\overline{p}_B(\overline{p}_S - \frac{\alpha_B}{\alpha_S}\,\overline{p}_B)$
II^b	$\langle 0, \overline{p}_B - \frac{\alpha_S}{\alpha_B}\,\overline{p}_S \rangle$	$2\frac{\alpha_S}{t}\,\overline{p}_S(\overline{p}_B - \frac{\alpha_S}{\alpha_B}\,\overline{p}_S)$
III	$\langle \overline{p}_S - \frac{t}{4}, \overline{p}_B - \frac{\alpha_S t}{4\alpha_B} \rangle$	$\frac{\alpha_S}{2}\left(\overline{p}_S + \overline{p}_B - \frac{(\alpha_S+\alpha_B)t}{4\alpha_B}\right)$
IV^a	$\langle \frac{(\alpha_S+\alpha_B)t}{2\alpha_B} - \overline{p}_B, \overline{p}_B - \frac{\alpha_S t}{4\alpha_B} \rangle$	$\frac{\alpha_S(\alpha_S+2\alpha_B)t}{8\alpha_B}$
IV^b	$\langle 0, \overline{p}_B - \frac{\alpha_S t}{4\alpha_B} \rangle$	$\frac{\alpha_S}{2}\left(\overline{p}_B - \frac{\alpha_S t}{4\alpha_B}\right)$

Table 5.2: Equilibrium fees and payoffs for the different regions in case $\alpha_S < \alpha_B$.

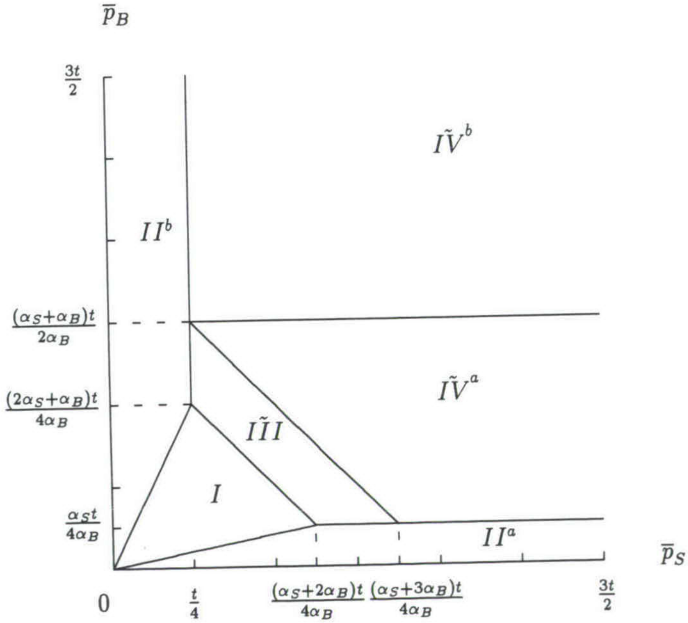

Figure 5.4: The different regions in case $\alpha_S < \alpha_B$.

We can distinguish three areas of no competition and three areas of weak competition. It is easily checked that the corresponding fees and payoffs change continuously in and between the areas, except between the areas II^a and IV^a where $\overline{p}_B = \frac{\alpha_S t}{4\alpha_B}$ and $\overline{p}_S > \frac{(\alpha_S+3\alpha_B)t}{4\alpha_B}$. There exists a continuum of equilibria at the intersection of areas II^a and IV^a for $\overline{p}_S > \frac{(\alpha_S+3\alpha_B)t}{4\alpha_B}$. Equilibrium fees are $\langle\varphi, 0\rangle$ and payoffs are $\frac{\alpha_S}{2}\varphi$ with $\varphi \in [\frac{(\alpha_S+2\alpha_B)t}{4\alpha_B}, \overline{p}_S - \frac{t}{4}]$. In equilibrium the buyers are served for free and the per middleman market size equals $\frac{\alpha_S}{2}$.

Areas I, II^a, II^b.

The areas I, II^a and II^b remain unchanged with respect to the case with equal densities.

Areas $\tilde{III}, \tilde{IV}^a, \tilde{IV}^b$.

Although area $\tilde{III}$ in Figure 5.4 has the same shape as area III in Figure 5.1, it is smaller, however. In area $\tilde{III}$, the sellers located at a distance $\frac{1}{4}$ from the middlemen have a zero surplus. A fraction $\alpha_B - \alpha_S$ of the buyers is not served. Firms do not try to capture them, since it is optimal to have equal shares.

In areas $\tilde{IV}^a$ and $\tilde{IV}^b$, the symmetric treatment of sellers and buyers disappears. Now, the reservation prices are so high, that the sellers located at a distance $\frac{1}{4}$ from both middlemen claim a positive surplus. They form the short side of the market and are able to take advantage of their position in the market. The negative effect of charging lower fees is more than compensated by the positive effect of attracting more sellers.

The advantageous market position of type 1 agents in case of high reservation prices is exercised maximally in area $\tilde{IV}^b$. Similar to the area II^b, the middlemen desire to subsidize the sellers agents. This results in them being served for free. The payoffs in $\tilde{IV}^b$ are increasing in the reservation price of the buyers, with no upper bound. Since competition on the long side of the market never occurs in equilibrium, the type buyers can be charged a very high price compared to their reservation price.

For the special case $\overline{p}_S = \overline{p}_B = \overline{p}$ equilibrium fees as a function of the reservation price $\overline{p}$ are drawn in Figure 5.5. We see that sellers are charged a lower fee than buyers for all values of reservation prices. Furthermore, sellers are served for free for relatively high values of the reservation price.

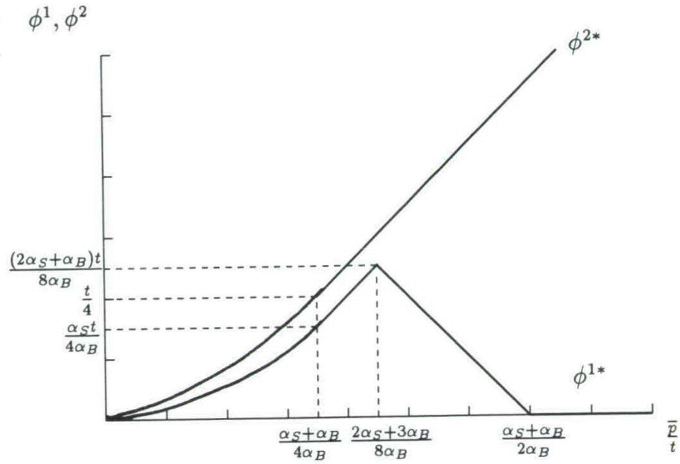

Figure 5.5: Equilibrium fees in case $\alpha_S < \alpha_B$.

We end this section with a comparison of equilibrium fees and payoffs for the cases of equal and unequal densities, respectively. For clarity we let $\overline{p}_S = \overline{p}_B = \overline{p}$.

If the reservation price is relatively low, i.e., $\overline{p} \leq \frac{(\alpha_S+4\alpha_B)t}{8\alpha_B}$, we are in regions I and III in case $\alpha_S = \alpha_B = \tilde{\alpha}_S$, and in regions $I, \tilde{III}$ and $\tilde{IV}^a$ in case $\tilde{\alpha}_S = \alpha_S < \alpha_B = \tilde{\alpha}_B$, For $\overline{p} \leq \frac{(\alpha_S+4\alpha_B)t}{8\alpha_B}$, payoffs are higher for the situation $\alpha_S < \alpha_B$ than for the situation $\alpha_S = \alpha_B$.

If the reservation price is relatively high, i.e., $\overline{p} \geq \frac{(\alpha_S+4\alpha_B)t}{4\alpha_B}$, we are in regions IV^a, IV^b and IV^c in case $\alpha_S = \alpha_B$, and in region. $\tilde{IV}^b$ in case $\alpha_S < \alpha_B$. For $\overline{p} \geq \frac{(\alpha_S+4\alpha_B)t}{4\alpha_B}$, payoffs are higher for the situation $\alpha_S < \alpha_B$ than for the situation $\alpha_S = \alpha_B$. Competition for the sellers becomes more severe in the latter case, which lowers payoffs.

If the reservation prices are intermediate, i.e., $\frac{(\alpha_S+4\alpha_B)t}{8\alpha_B} \leq \overline{p} \leq \frac{(\alpha_S+4\alpha_B)t}{4\alpha_B}$, payoffs are higher for the situation $\alpha_S = \alpha_B$ than for the situation $\alpha_S < \alpha_B$.

We conclude this section with a remark on the large indeterminacy found in case of equal densities, namely, the continua of equilibria. One could argue that the case of equal densities is non-generic. An arbitrary small disturbance on one side of the market, by adding or deleting agents, is sufficient to restore generic uniqueness of equilibrium. In this respect, a comparison can be made with Young (1993), who analyzes the evolution of bargaining over some amount of money between two agents of different types. Young finds several splits between types as equilibrium outcomes. If some agents change type sometimes, however, only an equal split remains as an equilibrium.

5.5 Proofs

We provide the derivation of the equilibria mentioned in Theorems 5.3.1 and 5.4.1, Propositions 5.3.2 and 5.4.2, and Lemma's 5.3.3 and 5.4.3 in this appendix. We do not refer explicitly to the separate results, since it is straightforward which result corresponds to which part of the derivation.

In order to prove the results we first specify the four relevant maximization problems. In the region of fees where there is no competition middleman j chooses fees ϕ_S^j and ϕ_B^j as to maximize

$$(\phi_S^j + \phi_B^j)\ min\left\{\frac{2\alpha_S}{t}(\overline{p}_S - \phi_S^j), \frac{2\alpha_B}{t}(\overline{p}_B - \phi_B^j)\right\} \tag{5.6}$$

subject to the constraints

$$\begin{array}{ll} \overline{p}_S \leq \frac{\phi_S^1+\phi_S^2}{2} + \frac{t}{4}, & 0 \leq \phi_S^j \leq \overline{p}_S \\ \overline{p}_B \leq \frac{\phi_B^1+\phi_B^2}{2} + \frac{t}{4}, & 0 \leq \phi_B^j \leq \overline{p}_B. \end{array}$$

In the region of fees where there is weak competition and the middlemen compete for sellers, middleman j chooses fees ϕ_S^j and ϕ_B^j, given ϕ^k of middleman $k \neq j$, as to maximize

$$(\phi_S^j + \phi_B^j)\ min\left\{\frac{\alpha_S}{t}(\phi_S^k - \phi_S^j + \frac{t}{2}), \frac{2\alpha_B}{t}(\overline{p}_B - \phi_B^j)\right\} \tag{5.7}$$

subject to the fee constraints

$$\begin{array}{ll} \overline{p}_S \geq \frac{\phi_S^1+\phi_S^2}{2} + \frac{t}{4}, & 0 \leq \phi_S^j \leq \overline{p}_S \\ \overline{p}_B \leq \frac{\phi_B^1+\phi_B^2}{2} + \frac{t}{4}, & 0 \leq \phi_S^j \leq \overline{p}_B. \end{array}$$

In the region of fees where there is weak competition and the middlemen compete for buyers middlemen choose fees ϕ_S^j and ϕ_S^j that maximize

$$(\phi_S^j + \phi_S^j)\ min\left\{\frac{2\alpha_S}{t}(\overline{p}_S - \phi_S^j), \frac{\alpha_B}{t}(\phi_B^k - \phi_B^j + \frac{t}{2})\right\} \tag{5.8}$$

subject to the constraints

$$\begin{array}{ll} \overline{p}_S \leq \frac{\phi_S^1+\phi_S^2}{2} + \frac{t}{4}, & 0 \leq \phi_S^j \leq \overline{p}_S \\ \overline{p}_B \geq \frac{\phi_B^1+\phi_B^2}{2} + \frac{t}{4}, & 0 \leq \phi_B^j \leq \overline{p}_B. \end{array}$$

In the regions of fees where there is strong competition middlemen j chooses fees ϕ_S^j and ϕ_B^j as to maximize

$$(\phi_S^j + \phi_B^j)\ min\left\{\frac{\alpha_S}{t}(\phi_S^k - \phi_S^j + \frac{t}{2}), \frac{\alpha_B}{t}(\phi_B^k - \phi_B^j + \frac{t}{2})\right\} \tag{5.9}$$

subject to the constraints

$$\begin{array}{ll} \overline{p}_S \geq \frac{\phi_S^1+\phi_S^2}{2} + \frac{t}{4}, & 0 \leq \phi_S^j \leq \overline{p}_S \\ \overline{p}_B \geq \frac{\phi_B^1+\phi_B^2}{2} + \frac{t}{4}, & 0 \leq \phi_B^j \leq \overline{p}_B. \end{array}$$

No competition

First consider the situation of no competition. Because the shares of buyers and sellers must be equal in equilibrium, we can substitute $\phi_B^j = \overline{p}_B - \frac{\alpha_S}{\alpha_B}\,\overline{p}_S + \frac{\alpha_S}{\alpha_B}\,\phi_S^j$ into maximization problem (5.6) for $j \in \{1,2\}$. Note that one of the constraints becomes redundant. If we denote the vector of Lagrange multipliers by $\lambda_j \in \mathbb{R}_+^5$, the corresponding Lagrangian for middleman j reads $\mathcal{L}_j(\phi_S^j, \lambda_j) = \left(\frac{\alpha_S+\alpha_B}{\alpha_B}\,\phi_S^j + \overline{p}_B - \frac{\alpha_S}{\alpha_B}\,\overline{p}_S\right)(2(\overline{p}_S - \phi_S^j)) - \lambda_{j1}(2\overline{p}_S - \phi_S^1 - \phi_S^2 - \frac{t}{2}) - \lambda_{j2}(\frac{\alpha_S}{\alpha_B}\,\overline{p}_S + \overline{p}_B - \frac{\alpha_S}{\alpha_B}\,\phi_S^j - \phi_B^k - \frac{t}{2}) - \lambda_{j3}(-\phi_S^j) - \lambda_{j4}(\phi_S^j - \overline{p}_S) - \lambda_{j5}(\overline{p}_S - \frac{\alpha_B}{\alpha_S}\,\overline{p}_B - \phi_S^j)$ with $k \neq j \in \{1,2\}$. Middleman j thus wants to maximize $\mathcal{L}_j(\phi_j^S, \lambda_j)$ with respect to ϕ_S^j and $\lambda_j \in \mathbb{R}_+^5$. The first order conditions for payoff maximization for middleman j can be written then as

$$\begin{cases} 2\left(\frac{2\alpha_S+\alpha_B}{\alpha_B}\right)\overline{p}_S - 2\overline{p}_B - 4\left(\frac{\alpha_S+\alpha_B}{\alpha_B}\right)\phi_S^j + \lambda_{j1} + \frac{\alpha_S}{\alpha_B}\lambda_{j2} + \lambda_{j3} - \lambda_{j4} + \lambda_{j5} = 0 \\ \lambda_{j1}(2\overline{p}_S - \phi_S^1 - \phi_S^2 - \frac{t}{2}) = 0 \\ \lambda_{j2}(\frac{\alpha_S}{\alpha_B}\,\overline{p}_S + \overline{p}_B - \frac{\alpha_S}{\alpha_B}\,\phi_S^j - \phi_B^k - \frac{t}{2}) = 0 \\ \lambda_{j3}(-\phi_S^j) = 0 \\ \lambda_{j4}(\phi_S^j - \overline{p}_S) = 0 \\ \lambda_{j5}(\overline{p}_S - \frac{\alpha_B}{\alpha_S}\,\overline{p}_B - \phi_S^j) = 0 \\ (2\overline{p}_S - \phi_S^1 - \phi_S^2 - \frac{t}{2}) \leq 0 \\ (\frac{\alpha_S}{\alpha_B}\,\overline{p}_S + \overline{p}_B - \frac{\alpha_S}{\alpha_B}\,\phi_S^j - \phi_B^k - \frac{t}{2}) \leq 0 \\ (-\phi_S^j) \leq 0 \\ (\phi_S^j - \overline{p}_S) \leq 0 \\ (\overline{p}_S - \frac{\alpha_B}{\alpha_S}\,\overline{p}_B - \phi_S^j) \leq 0 \\ \lambda_{jl} \geq 0, l \in \{1,2,3,4,5\}. \end{cases}$$

Due to symmetry the first order conditions are solved by $\phi^{j*} = \langle \phi_S^*, \phi_B^* \rangle$ for $j \in \{1,2\}$. Solving these equations we get

$$\phi_S^* = \phi_B^* = \begin{cases} \langle \frac{(2\alpha_S+\alpha_B)\overline{p}_S - \alpha_B\overline{p}_B}{2(\alpha_S+\alpha_B)}, \frac{(\alpha_S+2\alpha_B)\overline{p}_B - \alpha_S\overline{p}_S}{2(\alpha_S+\alpha_B)} \rangle & \text{if } \overline{p}_S + \overline{p}_B \leq \frac{(\alpha_S+\alpha_B)t}{2\alpha_B}, \\ & \quad \frac{\alpha_B}{2\alpha_S+\alpha_B}\,\overline{p}_B \leq \overline{p}_S \leq \frac{\alpha_S+2\alpha_B}{\alpha_S}\,\overline{p}_B \\ \langle 0, \overline{p}_B - \frac{\alpha_S}{\alpha_B}\,\overline{p}_S \rangle & \text{if } \overline{p}_S \leq \frac{\alpha_B}{2\alpha_S+\alpha_B}\,\overline{p}_B,\ \overline{p}_S \leq \frac{t}{4} \\ \langle \overline{p}_S - \frac{\alpha_B}{\alpha_S}\,\overline{p}_B, 0 \rangle & \text{if } \overline{p}_B \leq \frac{\alpha_S}{\alpha_S+2\alpha_B}\,\overline{p}_S,\ \overline{p}_B \leq \frac{\alpha_S t}{4\alpha_B} \\ \langle \overline{p}_S - \frac{t}{4}, \overline{p}_B - \frac{\alpha_S t}{4\alpha_B} \rangle & \text{if } \overline{p}_S + \overline{p}_B \geq \frac{(\alpha_S+\alpha_B)t}{2\alpha_B},\ \overline{p}_S \geq \frac{t}{4}, \\ & \quad \overline{p}_B \geq \frac{\alpha_S t}{4\alpha_B}. \end{cases}$$

Finally, we have to check whether or not (any of) these solutions can be improved upon. For all the solutions it holds that deviating by setting a higher fee for sellers (and consequently for buyers) decreases payoffs. The more interesting situation is deviating by setting a lower fee for sellers which of course cannot occur in case the other middleman charges fees $\langle 0, \overline{p}_B - \frac{\alpha_S}{\alpha_B}\,\overline{p}_S \rangle$. If the other middleman charges $\langle \overline{p}_S - \frac{\alpha_B}{\alpha_S}\,\overline{p}_B, 0 \rangle$, deviating by setting a lower fee for the sellers decreases payoffs, because demand cannot increase.

For the case $\alpha_S = \alpha_B$, if the other middleman charges fees $\langle \overline{p}_S - \frac{t}{4}, \overline{p}_B - \frac{t}{4} \rangle$, deviating by setting lower fees, say $\langle \overline{p}_S - \frac{t}{4} - \Delta, \overline{p}_B - \frac{t}{4} - \Delta \rangle$, yields payoffs $(\overline{p}_S + \overline{p}_B - \frac{t}{2} - 2\Delta)(\frac{t}{2} + \Delta)$. The derivative of these payoffs with respect to Δ is equal to $\overline{p}_S + \overline{p}_B - \frac{3t}{2} - 3\Delta$. So deviating is not optimal as long as $\overline{p}_S + \overline{p}_B \leq \frac{3t}{2}$.

If the other middleman charges fees $\langle \overline{p}_S - \frac{t}{4}, \overline{p}_B - \frac{t}{4} \rangle$, deviating by setting higher fees, say $\langle \overline{p}_S - \frac{t}{4} + \Delta, \overline{p}_B - \frac{t}{4} + \Delta \rangle$, yields payoffs $(\overline{p}_S + \overline{p}_B - \frac{t}{2} + 2\Delta)(\frac{t}{2} - 2\Delta)$. The derivative of these payoffs w.r.t. Δ is equal to $2t - 2\overline{p}_S - 2\overline{p}_B - 4\Delta$. This means that deviating by setting higher fees is not optimal as long as $\overline{p}_S + \overline{p}_B \geq t$.

The case $\alpha_S < \alpha_B$ differs from the case $\alpha_S = \alpha_B$ if the other middleman charges $\langle \overline{p}_S - \frac{t}{4}, \overline{p}_B - \frac{\alpha_S t}{4\alpha_B} \rangle$. Then, deviating by setting a lower fee for the sellers decreases payoffs as long as $\overline{p}_S + \overline{p}_B \leq \frac{(2\alpha_S + 3\alpha_B)t}{4\alpha_B}$. Finally, if the other middleman charges $\langle \frac{(2\alpha_S+\alpha_B)\overline{p}_S - \alpha_B \overline{p}_B}{2(\alpha_S+\alpha_B)}, \frac{(\alpha_S+2\alpha_B)\overline{p}_B - \alpha_S \overline{p}_S}{2(\alpha_S+\alpha_B)} \rangle$, deviating by setting a lower fee for the sellers decreases payoffs. For the solution $\phi^{1*} = \phi^{2*} = \langle \overline{p}_S - \frac{t}{4}, \overline{p}_B - \frac{\alpha_S t}{4\alpha_B} \rangle$ we thus have to impose the additional requirement that $\overline{p}_S + \overline{p}_B \leq \frac{(2\alpha_S + 3\alpha_B)t}{4\alpha_B}$.

Weak competition

Next, consider the situation of weak competition. Because shares have to be equal in equilibrium, we can substitute $\overline{p}_B - \frac{\alpha_S}{2\alpha_B}\left(\phi_S^k - \phi_S^j + \frac{t}{2}\right)$ for ϕ_B^j into maximization problem (5.7) for $j \neq k \in \{1,2\}$. We need not consider maximization problem (5.8) because $\alpha_S < \alpha_B$. If we denote the vector of Lagrange multipliers by $\lambda_j \in \mathbb{R}^6_+$, the corresponding Lagrangian for middleman j, reads $\mathcal{L}_j(\phi_S^j, \lambda_j) = \left(\frac{\alpha_S + 2\alpha_B}{2\alpha_B}\phi_S^j - \frac{\alpha_S}{2\alpha_B}\phi_S^k + \overline{p}_B - \frac{\alpha_S t}{4\alpha_B}\right)(\phi_S^k - \phi_S^j + \frac{t}{2}) - \lambda_{j1}(\phi_S^1 + \phi_S^2 + \frac{t}{2} - 2\overline{p}_S) - \lambda_{j2}(2\overline{p}_B - t + \frac{\alpha_S t}{2\alpha_B} - 2\phi_B^k + \frac{\alpha_S}{\alpha_B}(\phi_S^k - \phi_S^j)) - \lambda_{j3}(-\phi_S^j) - \lambda_{j4}(\phi_S^j - \overline{p}_S) - \lambda_{j5}(\phi_S^k + \frac{t}{2} - \frac{2\alpha_B}{\alpha_S}\,\overline{p}_B - \phi_S^j) - \lambda_{j6}(\phi_S^j - \phi_S^k - \frac{t}{2})$. Middleman j thus wants to maximize $\mathcal{L}_j(\phi_S^j, \lambda_j)$ with respect to ϕ_S^j and $\lambda_j \in \mathbb{R}^6_+$. The first order conditions for payoff maximization for middleman j can be written as

$$\begin{cases} -\overline{p}_B - \frac{\alpha_S+2\alpha_B}{\alpha_B}\phi_S^j + \frac{\alpha_S+\alpha_B}{\alpha_B}\phi_S^k + \frac{(\alpha_S+\alpha_B)t}{2\alpha_B} - \lambda_{j1} + \frac{\alpha_S}{\alpha_B}\lambda_{j2} + \lambda_{j3} - \lambda_{j4} + \lambda_{j5} - \lambda_{j6} = 0 \\ \lambda_{j1}(\phi_S^1 + \phi_S^2 + \frac{t}{2} - 2\overline{p}_S) = 0 \\ \lambda_{j2}(2\overline{p}_B - t + \frac{\alpha_S t}{2\alpha_B} - 2\phi_B^k + \frac{\alpha_S}{\alpha_B}(\phi_S^k - \phi_S^j)) = 0 \\ \lambda_{j3}(-\phi_S^j) = 0 \\ \lambda_{j4}(\phi_S^j - \overline{p}_S) = 0 \\ \lambda_{j5}(\phi_S^k + \frac{t}{2} - \frac{2\alpha_B}{\alpha_S}\,\overline{p}_B - \phi_S^j) = 0 \\ \lambda_{j6}(\phi_S^j - \phi_S^k - \frac{t}{2}) = 0 \\ (\phi_S^1 + \phi_S^2 + \frac{t}{2} - 2\overline{p}_S) \leq 0 \\ (2\overline{p}_B - t + \frac{\alpha_S t}{2\alpha_B} - 2\phi_B^k + \frac{\alpha_S}{\alpha_B}(\phi_S^k - \phi_S^j)) \leq 0 \\ (-\phi_S^j) \leq 0 \\ (\phi_S^j - \overline{p}_S) \leq 0 \\ (\phi_S^k + \frac{t}{2} - \frac{2\alpha_B}{\alpha_S}\,\overline{p}_B - \phi_S^j) \leq 0 \\ (\phi_S^j - \phi_S^k - \frac{t}{2}) \leq 0 \\ \lambda_{jl} \geq 0, l \in \{1,2,3,4,5,6\}. \end{cases}$$

Due to symmetry the first order conditions are solved by $\phi^{j*} = \langle \phi_S^*, \phi_B^* \rangle$ for $j \in \{1,2\}$. Solving these equations we get

$$\phi_S^* = \phi_B^* = \begin{cases} \langle 0, \overline{p}_B - \frac{\alpha_S t}{4\alpha_B} \rangle & \text{if } \overline{p}_B \geq \frac{(\alpha_S+\alpha_B)t}{2\alpha_B},\ \overline{p}_S \geq \frac{t}{4} \\ \langle \frac{(\alpha_S+\alpha_B)t}{2\alpha_B} - \overline{p}_B, \overline{p}_B - \frac{\alpha_S t}{4\alpha_B} \rangle & \text{if } \overline{p}_S + \overline{p}_B \geq \frac{(2\alpha_S+3\alpha_B)t}{4\alpha_B}, \\ & \quad \frac{\alpha_S t}{4\alpha_B} \leq \overline{p}_B \leq \frac{(\alpha_S+\alpha_B)t}{2\alpha_B} \\ \langle \overline{p}_S - \frac{t}{4}, \overline{p}_B - \frac{\alpha_S t}{4\alpha_B} \rangle & \text{if } \overline{p}_S + \overline{p}_B \leq \frac{(2\alpha_S+3\alpha_B)t}{4\alpha_B}, \\ & \quad \overline{p}_S \geq \frac{t}{4},\ \overline{p}_B \geq \frac{\alpha_S t}{4\alpha_B}. \end{cases}$$

Finally we have to check whether or not (any of) these solutions can be improved upon. As we have seen before we have to impose the additional requirement that $\overline{p}_S + \overline{p}_B \geq \frac{(\alpha_S+\alpha_B)t}{2\alpha_B}$ for the solution $\langle \overline{p}_S - \frac{t}{4}, \overline{p}_B - \frac{\alpha_S t}{4\alpha_B} \rangle$.

Strong competition

Consider the situation of strong competition. Because shares have to be equal in equilibrium, we can substitute $\phi_B^j = \phi_B^k + \phi_S^j - \phi_S^k$ into maximization problem (5.9) for $j \neq k \in \{1,2\}$. If we denote the vector of Lagrange multipliers by $\lambda_j \in \mathbb{R}_+^6$, the corresponding Lagrangian for middleman j, $j \in \{1,2\}$, reads $\mathcal{L}_j(\phi_S^j, \lambda_j) = (2\phi_S^j + \phi_B^k - \phi_S^k)(\phi_S^k - \phi_S^j + \frac{t}{2}) - \lambda_{j1}(-\phi_S^j) - \lambda_{j2}(\phi_S^j - \overline{p}_S) - \lambda_{j3}(\phi_S^k - \phi_B^k - \phi_S^j) - \lambda_{j4}(\phi_S^j + \phi_B^k - \phi_S^k -$

$\overline{p}_B) - \lambda_{j5}(\phi_S^1 + \phi_S^2 + \frac{t}{2} - 2\overline{p}_S) - \lambda_{j6}(2\phi_B^k + \phi_S^j - \phi_S^k + \frac{t}{2} - 2\overline{p}_B)$. The first order conditions for payoff maximization for middleman j, $j \in \{1,2\}$, can be written then as

$$\begin{cases} 3\phi_S^k - \phi_B^k - 4\phi_S^j + t + \lambda_{j1} - \lambda_{j2} + \lambda_{j3} - \lambda_{j4} - \lambda_{j5} - \lambda_{j6} = 0 \\ \lambda_{j1}(-\phi_S^j) = 0 \\ \lambda_{j2}(\phi_S^j - \overline{p}_S) = 0 \\ \lambda_{j3}(\phi_S^k - \phi_B^k - \phi_S^j) = 0 \\ \lambda_{j4}(\phi_S^j + \phi_B^k - \phi_S^k - \overline{p}_B) = 0 \\ \lambda_{j5}(\phi_S^1 + \phi_S^2 + \frac{t}{2} - 2\overline{p}_S) = 0 \\ \lambda_{j6}(2\phi_B^k + \phi_S^j - \phi_S^k + \frac{t}{2} - 2\overline{p}_B) = 0 \\ (-\phi_S^j) \leq 0 \\ (\phi_S^j - \overline{p}_S) \leq 0 \\ (\phi_S^k - \phi_B^k - \phi_S^j) \leq 0 \\ (\phi_S^j + \phi_B^k - \phi_S^k - \overline{p}_B) \leq 0 \\ (\phi_S^1 + \phi_S^2 + \frac{t}{2} - 2\overline{p}_S) \leq 0 \\ (2\phi_B^k + \phi_S^j - \phi_S^k + \frac{t}{2} - 2\overline{p}_B) \leq 0 \\ \lambda_{jl} \geq 0, l \in \{1,2,3,4,5,6\}. \end{cases}$$

Due to symmetry the first order conditions are solved by $\phi^{j*} = \langle \phi_S^{j*}, \phi_B^{j*} \rangle$ for $j \in \{1,2\}$. Solving these equations we get

$$\phi_S^* = \phi_B^* = \begin{cases} \langle \varphi, t-\varphi \rangle & \text{if } \ 0 \leq \varphi \leq \overline{p}_S - \frac{t}{4},\ \frac{5t}{4} - \overline{p}_B \leq \varphi \leq t \\ \langle 0, \varphi \rangle & \text{if } \ t \leq \varphi \leq \overline{p}_B - \frac{t}{4} \\ \langle \varphi, 0 \rangle & \text{if } \ t \leq \varphi \leq \overline{p}_S - \frac{t}{4} \\ \langle \overline{p}_S - \frac{t}{4}, \overline{p}_B - \frac{t}{4} \rangle & \text{if } \ \overline{p}_S + \overline{p}_B \leq \frac{3t}{2} \end{cases}$$

where $\overline{p}_S \geq \frac{t}{4}$ and $\overline{p}_B \geq \frac{t}{4}$.

Finally, we have to check whether or not (any of) these solutions can be improved upon.

Recall that any solution $\phi_S^* = \phi_B^* = \langle \mu, \nu \rangle$ to (5.9) satisfies $0 \leq \mu \leq \overline{p}_S - \frac{t}{4}$ and $0 \leq \nu \leq \overline{p}_B - \frac{t}{4}$. First consider the situation where $0 < \mu < \overline{p}_S - \frac{t}{4}$ and $0 < \nu < \overline{p}_B - \frac{t}{4}$. If a middleman deviates by setting slightly lower fees, say $\mu - \Delta$ and $\nu - \Delta$ for some $\Delta > 0$, payoffs are $(\mu + \nu - 2\Delta)(\frac{t}{2} + \Delta)$. The derivative of these payoffs with respect to fees is equal to $\mu+\nu-\gamma_1-4\Delta$, so deviating by setting lower fees is not optimal as long as $\mu+\nu\leq t$. Similarly we find that deviating by setting higher fees is not optimal as long as $\mu+\nu\geq t$. Combining these results gives that $\mu+\nu=t$. If fees increase more, the situation of no competition occurs. This requires that $\Delta\geq\Delta^*=2\overline{p}_S-\frac{t}{2}-2\mu$. Payoffs are equal then to $2(2\mu+\overline{p}_B-\overline{p}_S-2\Delta)(\overline{p}_S-\mu-\Delta)$. One can check that the derivative of these payoffs is negative at Δ^*, so deviating to the situation of no competition cannot be optimal. Next consider the situation where one of the two fees is zero. Then we need only consider deviations by setting higher fees. As shown before this means that $\mu+\nu\geq t$. Note that the situation where both fees are zero cannot occur. Finally consider the situation where $\mu=\overline{p}_S-\frac{t}{4}$ and (consequently) $\nu=\overline{p}_B-\frac{t}{4}$. As shown, this can only be Nash as long as $t\leq\overline{p}_S+\overline{p}_B\leq\frac{3t}{2}$.

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Samenvatting

Voor het goed functioneren van markten is in de meeste gevallen de aanwezigheid van een coördinerende instantie noodzakelijk. Hierbij kan men denken aan traditionele instituten als banken, makelaars en veilingen, maar ook aan de 'elektronische snelweg' als intermediair tussen marktpartijen. In het algemeen worden markten 'geactiveerd' doordat bepaalde groepen agenten de behoefte van andere agenten herkennen om te handelen. In ruil voor het activeren van markten is deze laatste groep, de handelaren, bereid om een deel van hun handelswinst over te dragen aan de eerste groep, de intermediairs. Zodoende is de markt voor alle agenten profijtelijk.

Het traditionele model ter beschrijving van de werking van markten is het Walrasiaanse marktmodel. Dit model geeft een wiskundige formalisatie van het principe van de 'onzichtbare hand' zoals dat werd geïntroduceerd door Adam Smith. De agenten in dit model gedragen zich als volledige mededingers, dat wil zeggen, alsof ze geen invloed hebben op de prijs. Ofschoon het Walrasiaanse marktmodel helder en wiskundig elegant is, kleven er bezwaren aan die vanaf het ontstaan van dit model reeds onderkend zijn. Een van deze bezwaren is de afwezigheid van een expliciete organisatorische structuur. Traditioneel wordt een dergelijke organisatie geïntroduceerd in de vorm van een 'marktmeester' die de marktprijs vaststelt. Deze marktmeester is echter impliciet in de zin dat hij geen agent is. Het eerste deel van deze monografie voegt een expliciete organisatiestructuur toe aan het Walrasiaanse marktmodel middels een coalitie van expliciete marktmeesters. Deze coalitie wordt endogeen bepaald. In het tweede deel van het proefschrift bekijken we een model waarin de rol van marktmeester exogeen wordt toebedeeld.

Na het inleidende hoofdstuk 1, waarin de verschillende modellen worden toegelicht, volgt het eerste deel van de monografie, dat de hoofdstukken 2, 3 en 4 beslaat. De modellen in deze hoofdstukken beschouwen goederenmarkten die geactiveerd worden door intermediairs en gebruikt worden door handelaren. De rol van intermediair en handelaar is vrij toegankelijk voor alle consumenten. De keuze die iedere consument maakt is afhankelijk van de kosten en baten van een rol en markt.

In hoofdstuk 2 wordt het marktformatie-spel in zijn algemene vorm geïntroduceerd. Het marktformatie-spel verbindt de individuele met de institutionele karakteristieken van een economie. De individuele kant wordt gevormd door consumenten met hun karakteristieken, namelijk beginvoorraden en nutsfuncties; dit is de traditionele Arrow/Debreu economie. De institutionele kant wordt gevormd door een collectie lokale markten die worden 'geactiveerd' door intermediairs en gebruikt door handelaren. De markten zijn lokaal in de zin dat de goederenbundels die op een bepaalde markt verhandeld worden uitsluitend worden bepaald door de samenstelling van de groepen handelaren die aanwezig zijn op die markt. Hierin verschillen de markten in het marktformatie-model van de traditionele Walrasiaanse markten; daar is sprake van één globale markt. Een ander verschil met het Walrasiaanse marktmodel is gelegen in het feit dat markten kostendragend zijn in de zin dat zowel het activeren van een markt door intermediairs als het gebruik van een markt door handelaren kosten met zich meebrengt. De kosten voor intermediairs worden veroorzaakt door het opzetten van een markt, en door het niet kunnen handelen met hun beginvoorraad (*opportunity costs*). De kosten voor handelaren worden gegeven door de toegangsprijs van een markt, die tevens een vergoeding voor de intermediairs op die markt is.

Rationeel gedrag van de consumenten in het marktformatie-spel leidt ertoe dat rollen en markten geselecteerd worden zolang zij meer opleveren dan andere, totdat een Nash-evenwicht is bereikt. Een Nash-evenwicht is een verdeling van consumenten over rollen en markten waarin geen consument erop vooruit gaat door een andere rol en markt te kiezen. De vrije toegankelijkheid van rollen en markten voor alle consumenten creëert in zekere zin een situatie van volledige mededinging voor de formatie van een organisatie-structuur.

Het algemene model in hoofdstuk 2 levert een zeer groot aantal Nash-evenwichten op. In principe kan een willekeurig aantal actieve markten met voldoende kleine toegangsprijs ondersteund worden. Dit wordt veroorzaakt door de markt-externaliteit gerelateerd aan een continuüm van consumenten: individuele consumenten zijn niet in staat om te handelen. Nog een andere externaliteit wordt gevonden, namelijk het onvermogen van individuele intermediairs om markten te activeren. Deze twee externaliteiten hebben tot gevolg dat consumenten gemakkelijk 'ingesloten' worden in onaantrekkelijke markten, bijvoorbeeld markten met een hoge toegangsprijs. Ofschoon dan de behoefte bestaat om een markt met een lage toegangsprijs te activeren, zijn individuele consumenten hiertoe niet in staat. Coalities van consumenten hebben die mogelijkheid echter in principe wel.

In hoofdstuk 2 en 3 worden de markt-externaliteiten 'opgelost' door de mogelijkheid tot afwijken door coalities te beschouwen. Het hierbij behorende evenwichtsconcept is het zogenaamde 'sterke' evenwicht, wat gezien kan worden als het coalitionele analogon van het Nash-evenwicht. Een verdeling van consumenten over markten en rollen is een sterk evenwicht indien er geen coalitie van consumenten bestaat die door een alternatieve keuze van markten en rollen al haar leden een hogere uitbetaling geeft dan in de oorspronkelijke situatie.

Het blijkt niet eenvoudig te zijn om algemene uitspraken te doen over de structuur van de verzameling sterke evenwichten. Dit wordt veroorzaakt door de afhankelijkheid van de ruilopbrengsten met betrekking tot de samenstelling van consumenten-typen op een markt; een afwijking naar een markt met een lagere toegangsprijs hoeft niet automatisch een verbetering te betekenen. Voor het meer specifieke model van hoofdstuk 3 wordt wel als resultaat gevonden dat een coalitie profijtelijk kan afwijken naar een markt met een lagere toegangsprijs. In dat hoofdstuk wordt een een economie met twee goederen en twee markten beschouwd, die geografisch van elkaar gescheiden zijn. De transportkosten tussen de markten zijn van invloed op de structuur van evenwichtsverdelingen; relatief lage transportkosten genereren evenwichten waarbij slechts één markt actief is, terwijl bij relatief hoge transportkosten beide markten actief zijn.

Het bekijken van coalitionele afwijkingen in hoofdstuk 2 en 3 gaat uit van een verhoogde graad van rationaliteit van de consumenten. Niet alleen worden consumenten geacht het effect van hun individuele afwijkingen te kunnen inschatten, maar ook het effect van iedere afwijking door een coalitie waar zij deel van uitmaken. In hoofdstuk 4 wordt de graad van rationaliteit verlaagd; in dat hoofdstuk zijn de consumenten begrensd rationeel. In plaats van een volledige calculatie van de effecten van hun keuzes kiezen de consumenten rollen en markten op basis van hun relatieve winstgevendheid. In een dynamisch model, waarin het marktformatie-spel steeds opnieuw wordt gespeeld, worden rollen en markten die in het verleden hoge uitbetalingen opleverden door een groter aantal consumenten gekozen. Beschouwd wordt nu de evolutionaire stabiliteit van Nash-evenwichten. Evolutionair stabiele verdelingen van consumenten over markten en rollen hebben de eigenschap dat ze in de tijd niet meer veranderen, en bovendien ongevoelig zijn voor relatief kleine verstoringen van de verdeling. Het model in hoofdstuk 4 is analoog aan dat van hoofdstuk 3 en genereert gelijksoortige evenwichten. Van de evenwichten met slechts één actieve markt blijkt het evenwicht waarin relatief 'goedkope' markt actief is, in termen van toegangsprijs en transportkosten, evolutionair stabiel te zijn. Verder is een evenwicht met twee actieve markten altijd evolutionair stabiel.

Deel 2 van de monografie wordt gevormd door hoofdstuk 5. Daarin wordt een model geanalyseerd waarin twee marktpartijen bilateraal wensen te handelen. Dat wil zeggen dat iedere agent van de ene partij een transactie zoekt met een agent van de andere partij. Het koppelen van agenten wordt gedaan door twee intermediairs. Een voorbeeld van een dergelijke markt is de huizenmarkt. Daar wensen verkopers van huizen gekoppeld te worden met kopers, waarbij het koppelingsproces plaatsvindt via makelaars. Een ander voorbeeld is de huwelijksmarkt.
De intermediairs zijn exogeen bepaald. Dit in tegenstelling tot de modellen in het eerste deel waarbij de coalitie van intermediairs endogeen bepaald wordt in het marktformatiespel. De intermediairs rekenen een commissie voor het bij elkaar brengen van de marktpartijen. De commissies worden gekozen in een niet-coöperatief prijsspel volgens een symmetrisch Nash-evenwicht.
Net als in de modellen in het eerste deel is hier sprake van een externaliteit: de winst van een intermediair wordt bepaald door het minimum van zijn aandelen van de twee marktkanten. Een interessant resultaat van het model, gerelateerd aan deze externaliteit, is dat mogelijk een van de marktpartijen gratis gebruik maakt van de koppel-service van de intermediairs. In dat geval wordt hun winst geheel bepaald door de andere marktpartij.

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Market Formation and Market Selection

van Chris van Raalte.

I

Het model dat geanalyseerd wordt in Hoofdstuk 5 kan door entertainers die proberen optredens te krijgen gebruikt worden om theaterburo's over te halen hen te subsidiëren.

[Zie: Proposition 5.3.2, p. 110 en Proposition 5.4.2, p. 114]

II

De heilige boeken van de grote wereldgodsdiensten bevatten informatie over de menselijke psychologie en zijn als zodanig een leidraad voor het functioneren van individuen en samenlevingen.

[Zie ook: SKYNNER, R. AND J. CLEESE (1993), *Life and how to survive it*, Methuen, London.]

III

De stelling dat verlegenheid een ziekte is leidt niet tot tegenspraak.

IV

Het toekennen van een morele waarde aan zaken als efficiëntie en werklust, kenmerkend voor kapitalistische samenlevingen, is een poging de onzekerheid die voortvloeit uit (economische) vrijheid niet te hoeven ervaren.

[Zie ook: FROMM, E. (1942), *The Fear of Freedom*, Paul Kegan, London.]

V

De beste manier om bij het zingen van een baspartij geen last met de buren te krijgen is zoveel volume te produceren dat de deurbel niet gehoord wordt.

VI

Een pijnervaring die niet emotioneel wordt ontladen leidt tot een afname van intelligentie in situaties die met een dergelijke pijnervaring worden geassocieerd.

[Zie ook: JACKINS, H. (1978), *The Human Side of Human Beings: The Theory of Re-evaluation Counseling*, Rational Island Publishers, Seattle.]

VII

De in duursporten dikwijls verwisselde termen 'afzien' en 'stukzitten' duiden zeer verschillende fenomenen aan.

VIII

Zelfverzekerdheid op het toneel is alleen leuk als een speler daarmee tevergeefs zijn eigen onzekerheid tracht te maskeren.

IX

Door het gebruik van 'humane' terechtstellingsmethoden zoals de dodelijke injectie wordt de voltrekking van een doodvonnis min of meer hetzelfde als een medische handeling. Hierdoor wordt het buitengewoon wrede karakter van de doodstraf minder snel onderkend.

[Zie ook: TROMBLEY, S. (1993), *The Execution Protocol*, Century, London.]

X

Aan autisme verwante karaktertrekken die zich uiten in 'vreemd' sociaal gedrag zijn mogelijk tevens verantwoordelijk voor uitingen van meer dan gemiddelde creativiteit. De eugenetici die zich tot doel stellen de genen die ten grondslag liggen aan dergelijke karaktertrekken te elimineren, bewijzen een samenleving geen dienst.

[Zie ook: SACKS, O. (1995), *An Anthropologist on Mars*, Picador, London.]

XI

Wie inziet dat hij nooit een Nobelprijs zal winnen, leert een Zweedse kennen.

CHRIS VAN RAALTE graduated at Tilburg University in 1992. Consequently, he has been working as a Ph.D. student at the Department of Econometrics at the same university. From 1993 to 1994 he visited the Department of Economics of Virginia Polytechnic Institute and State University, Blacksburg, Virginia, U.S.A.

The organization of markets is an important field of inquiry in modern economic theory. This monograph analyzes models which consider the formation and selection of markets. In these models, markets are organized by middlemen and used by traders. In Part I of the monograph, coalitions of middlemen are determined endogenously. Arrow/Debreu type consumers choose roles (middleman or trader) and markets in a non-cooperative market formation game. Distributions of consumers over roles and markets that are individually as well as coalitionally stable are investigated. In Part II, the competition in commission fees between exogenously given middlemen, who intermediate on a bilateral matching market is analyzed.

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